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A study on Fekete-Szegö inequality for a class of analytic functions satisfying subordinate condition associated with Chebyshev polynomials

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Abstract. We define a class of analytic functions, $A(H, n, m, \lambda)$, satisfying the following condition

$$\frac{D_\lambda^{n+m} f(z)}{D_\lambda^n f(z)} \prec H(z, t),$$

where $\lambda \geq 0, n, m \in \mathbb{N}^* = \mathbb{N} \cup \{0\}, t \in \left(\frac{1}{2}, 1\right]$ and for all $z \in \Omega$. In this study, firstly give estimates for coefficients $|a_2|$ and $|a_3|$ of functions belong to this class. Furthermore, the Fekete- Szegö inequality was examined for the functions belonging to this class.

1. Introduction

Let A denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1.1}$$

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which are analytic in the open unit disk $\Omega = \{z \in C : |z| < 1\}$. Let A_j be the family of functions of the form

$$f(z) = z + \sum_{k=j+1}^{\infty} a_k z^k, \quad (j \in \mathbb{N} = \{1, 2, 3, \ldots\}) \tag{1.2}$$

which are analytic in the open unit disk Ω. For a function $f(z) \in A_j$, we define

$$D^0 f(z) = f(z), D^1 f(z) = Df(z) = zf'(z), ...$$

and

$$D^n f(z) = D\left(D^{n-1} f(z)\right) \quad (n \in \mathbb{N}).$$

The differential operator D^n was introduced by Salagean [14]. With the help of the differential operator D^n, we say that a function $f(z) \in A_j$ is in the class $A_j(n, m, \alpha)$ if and only if

$$Re\left\{\frac{D^{n+m} f(z)}{D^n f(z)}\right\} > \alpha \quad (n \in \mathbb{N}^* = \mathbb{N} \cup \{0\}, m \in \mathbb{N})\, \lambda \geq 0\Big) \tag{1.3}$$

for some α $(0 \leq \alpha < 1)$, and for all $z \in \Omega$. The operator D^{n+m} was introduced by Sekine [15], Aouf et al. [4].

In [3], AL-Oboudi defined the generalized Salagean operator as following:

Let $n \in \mathbb{N}^*$ and $\lambda \geq 0$. We let D^n_λ denote the operator defined by

$$\begin{gathered}
D^n_\lambda : A \to A, \\
D^0_\lambda f(z) = f(z), \\
D^1_\lambda f(z) = (1-\lambda) D^0_\lambda f(z) + \lambda z \left(D^0_\lambda f(z)\right)' = (1-\lambda) f(z) + \lambda z f'(z), \\
\cdots \\
D^{n+1}_\lambda f(z) = (1-\lambda) D^n_\lambda f(z) + \lambda z \left(D^n_\lambda f(z)\right)'.
\end{gathered}$$

When $\lambda = 1$, we get Salagean's differential operator [14]. For $f \in A_j(n, m, \alpha)$ using the definition of the D^n_λ operator, we write [6]

$$D^n_\lambda f(z) = z + \sum_{k=j+1}^{\infty} [1 + \lambda(k-1)]^n a_k z^k \tag{1.4}$$

With the above operator D^n_λ, we say that a function $f(z)$ belonging to A_j is in the class $A_j(n, m, \lambda, \alpha)$ if and only if

$$Re\left[\frac{D^{n+m}_\lambda f(z)}{D^n_\lambda f(z)}\right] > \alpha \quad (n, m \in \mathbb{N}^*, \lambda \geq 0) \tag{1.5}$$

for some $\alpha, (0 \leq \alpha < 1)$, and for all $z \in \Omega$. Let f and g be analytic functions in Ω. We define that the function f is subordinate to g in and denoted by

$$f(z) \prec g(z) \quad (z \in \Omega)$$

if there exists a Schwarz function ω, which is analytic in Ω with $\omega(0) = 0$ and $|\omega(z)| < 1$ $(z \in \Omega)$ such that

$$f(z) = g(\omega(z)) \quad (z \in \Omega).$$

If g is a univalent function in Ω, then

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(\Omega) \subset g(\Omega).$$

In 1933, Fekete and Szegö [9] obtained a sharp bound of the functional $a_3 - \mu a_2^2$, with real $\mu (0 \leq \mu \leq 1)$ for a univalent function f. Since then, the problem of finding the sharp bounds for this functional of any compact family of functions or $f \in A$ with any complex μ is known as the classical Fekete-Szegö problem or inequality.

Chebyshev polynomials have greater importance in numerical analysis and more generally in applications of Mathematics. There are four kinds of Chebyshev polynomials. The majority of books and research papers dealing with specific ortogonal polynomials of the Chebyshev family contain mainly results of Chebyshev polynomials of the first and second kinds $T_n(t), U_n(t)$ and their numerous uses in different applications; see, for example, Doha [7] and Mason [10].

The Chebyshev polynomials of the first and second kinds are well known. In the case of a real variable t on $(-1, 1)$, they are defined by

$$\begin{aligned} T_n(t) &= \cos n\varphi \\ U_n(t) &= \tfrac{\sin(n+1)\varphi}{\sin\varphi} \end{aligned}$$

where n denotes the polynomial degree and $t = \cos\varphi$. For a brief history of Chebyshev polynomials of the first kind $T_n(t)$, the second kind $U_n(t)$ and their applications one can refer ([1],[17]). We consider that if $t = \cos\varphi \left(\frac{-\pi}{3} < \varphi < \frac{\pi}{3}\right)$, then

$$H(z,t) := \frac{1}{1 - 2tz + z^2} = \frac{1}{1 - 2\cos\varphi z + z^2} = 1 + \sum_{n=1}^{\infty} \frac{\sin(n+1)\varphi}{\sin\varphi} z^n \quad (z \in \Omega).$$

Thus,

$$H(z,t) = 1 + 2\cos\varphi z + \left(3\cos^2\varphi - \sin^2\varphi\right) z^2 + \cdots \quad (z \in \Omega).$$

So, according to [17], we write the Chebyshev polynomials of the second kind as following:

$$H(z,t) = 1 + U_1(t)z + U_2(t)z^2 + \cdots \quad (z \in \Omega, -1 < t < 1) \tag{1.6}$$

where $U_{n-1}(t)=\frac{\sin(n\arccos t)}{\sqrt{1-t^2}}(n\in\mathbb{N})$ and we have

$$\begin{aligned} U_n(t) &= 2tU_{n-1}(t)-U_{n-2}(t)\\ U_0(t) &= 1\\ U_1(t) &= 2t\\ U_2(t) &= 4t^2-1\\ U_3(t) &= 8t^3-4t\\ U_4(t) &= 16t^4-12t^2+1,\cdots \end{aligned} \tag{1.7}$$

The Chebyshev polynomials $T_n(t), t\in[-1,1]$ of the first kind have the generating function of the form

$$\sum_{n=0}^{\infty}T_n(t)z^n=\frac{1-tz}{1-2tz+z^2}\quad (z\in\Omega).$$

There is the following connection by the Chebyshev polynomials of the first kind $T_n(t)$ and the second kind $U_n(t)$:

$$\frac{dT_n(t)}{dt}=nU_{n-1}(t),\quad T_n(t)=U_n(t)-tU_{n-1}(t),\quad 2T_n(t)=U_n(t)-U_{n-2}(t)$$

In 2015, Dziok et al. [8] have studied the coefficient bounds and Fekete-Szegö inequality for the function $f\in H(t), t\in\left(\frac{1}{2},1\right]$ satisfying the following condition

$$1+\frac{zf''(z)}{f'(z)}\prec H(z,t).$$

It is to be noted that $H(z,t)$ is not univalent in Ω and allows only limited considerations of the $H(z,t)$. In 2016, Altinkaya et al. [1] have defined and studied the coefficient estimates, Fekete-Szegö inequality for the functions $f\in K(\lambda,t), t\in\left(\frac{1}{2},1\right], \lambda\geq 0$, satisfying the following condition

$$(1-\lambda)\frac{zf'(z)}{f(z)}+\lambda\left(1+\frac{zf''(z)}{f'(z)}\right)\prec H(z,t).$$

In 2018, Altinkaya et al. [2] have defined and studied the coefficient estimates, Fekete-Szegö inequality for the functions $f\in L(\alpha,t), \alpha\geq 0, t\in\left(\frac{1}{2},1\right]$ satisfying the following condition

$$\left(\frac{zf'(z)}{f(z)}\right)^{\alpha}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\alpha}\prec H(z,t)\quad (z\in\Omega).$$

In 2019, Szatmari et al. [16] have defined and studied the coefficient estimates, Fekete-Szegö inequality for the functions $f\in F(H,\alpha,\delta,\mu),\quad 0\leq\alpha\leq 1,\quad 1\leq\delta\leq 2,\quad 0\leq\mu\leq 1, t\in\left(\frac{1}{2},1\right]$ satisfying the following condition

$$\left[\alpha\left(\frac{zf'(z)}{f(z)}\right)^{\delta}+(1-\alpha)\left(\frac{zf'(z)}{f(z)}\right)^{\mu}\left(1+\frac{zf''(z)}{f'(z)}\right)^{1-\mu}\right]\prec H(z,t)\quad(z\in\Omega).$$

In 2020, Çağlar et al. [5] have defined and studied the coefficient estimates, Fekete-Szegö inequality for the functions $f\in N(\lambda,\beta,t), 0\le\beta\le\lambda\le 1\quad t\in\left(\frac{1}{2},1\right]$ satisfying the following condition

$$\frac{\lambda\beta z^3 f'''(z)+(2\lambda\beta+\lambda-\beta)z^2 f''(z)+zf'(z)}{\lambda\beta z^2 f''(z)+(\lambda-\beta)zf'(z)+(1-\lambda+\beta)f(z)}\prec H(z,t)=\frac{1}{1-2tz+z^2}\quad(z\in\Omega).$$

Many studies have been conducted on different classes defined by many mathematicians on different dates and various results have been obtained([11, 12, 13]). Now, we define a subclass of analytic functions in Ω with the following subordination condition:

Definition 1.1. We say that Let $f\in A$ of the form (1.1) belongs to $A(H,n,m,\lambda)$ if

$$\frac{D_{\lambda}^{n+m}f(z)}{D_{\lambda}^{n}f(z)}\prec H(z,t)=\frac{1}{1-2tz+z^2},\tag{1.8}$$

where $\lambda\ge 0, n,m\in\mathbb{N}^*=\mathbb{N}\cup\{0\}, t\in\left(\frac{1}{2},1\right]$ and for all $z\in\Omega$.

In this paper, we obtain initial coefficients $|a_2|$ and $|a_3|$ for subclass $A(H,n,m,\lambda)$ by means of Chebyshev polynomials expansions of analytic functions in Ω. Also, we solve Fekete-Szegö problem for functions in this subclass.

2. Coefficients bounds and Fekete-Szegö problem for the functions belong to subclass $A(H,n,m,\lambda)$.

Theorem 2.1. *Let $f\in A$ of the form (1.1) belong to the class $A(H,n,m,\lambda)$. Then*

$$|a_2|\le\frac{2t}{(1+\lambda)^n\left[(1+\lambda)^m-1\right]}\tag{2.1}$$

and

$$|a_3|\le\frac{1}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\left\{\frac{4(1+\lambda)^m}{\left[(1+\lambda)^m-1\right]}t^2+2t-1\right\}.\tag{2.2}$$

Proof. Let $f\in A(H,n,m,\lambda)$, then from (1.6) and (1.8), we have

$$\frac{D_{\lambda}^{n+m}f(z)}{D_{\lambda}^{n}f(z)}=1+u_1(t)w(z)+u_2(t)w^2(z)+\cdots\tag{2.3}$$

for some analytic function w such that Let $w(0)=0$ and $|w(z)|<1$ for all and $z\in\Omega$. Thus, we can write

$$\frac{D_{\lambda}^{n+m}f(z)}{D_{\lambda}^{n}f(z)}=1+u_1(t)w_1z+\left(u_2(t)w_1^2+u_1(t)w_2\right)z^2+\cdots\tag{2.4}$$

It is fairly well-known that if $|w(z)| = |w_1 z + w_2 z^2 + w_3 z^3 + \cdots| < 1, z \in \Omega$, then

$$|w_j| \leq 1, \quad \text{for all } j \in \mathbb{N}; \tag{2.5}$$

and

$$\left|w_2 - \zeta w_1^2\right| \leq \max\{1, |\zeta|\}, \text{ for all } \zeta \in \mathbb{R} \text{ [1]}. \tag{2.6}$$

It follows from (2.4) that

$$\begin{aligned} & z + (1+\lambda)^{n+m} a_2 z^2 + (1+2\lambda)^{n+m} a_3 z^3 + \cdots \\ = & \left\{z + (1+\lambda)^n a_2 z^2 + (1+2\lambda)^n a_3 z^3 + \cdots\right\} \\ & \left\{1 + u_1(t) w_1 z + \left(u_2(t) w_1^2 + u_1(t) w_2\right) z^2 + \cdots\right\} \end{aligned}$$

Then,

$$\begin{aligned} & (1+\lambda)^{n+m} a_2 z^2 + (1+2\lambda)^{n+m} a_3 z^3 + \cdots = \left[(1+\lambda)^n a_2 + u_1(t) w_1\right] z^2 + \\ & \left[u_2(t) w_1^2 + u_1(t) w_2 + (1+\lambda)^n a_2 u_1(t) w_1 + (1+2\lambda)^n a_3\right] z^3 + \cdots \end{aligned}$$

and

$$(1+\lambda)^{n+m} a_2 = (1+\lambda)^n a_2 + u_1(t) w_1, \tag{2.7}$$

and

$$(1+2\lambda)^{n+m} a_3 = u_2(t) w_1^2 + u_1(t) w_2 + (1+\lambda)^n a_2 u_1(t) w_1 + (1+2\lambda)^n a_3. \tag{2.8}$$

From (1.7), (2.5) and (2.7), we obtain

$$(1+\lambda)^{n+m} a_2 = (1+\lambda)^n a_2 + u_1(t) w_1,$$

and so,

$$a_2 = \frac{u_1(t) w_1}{(1+\lambda)^n \left[(1+\lambda)^m - 1\right]}.$$

This implies that

$$|a_2| \leq \frac{2t}{(1+\lambda)^n \left[(1+\lambda)^m - 1\right]}. \tag{2.9}$$

Similarly, by (1.7), (2.5) and (2.8), we write

$$(1+2\lambda)^{n+m} a_3 = u_2(t) w_1^2 + u_1(t) w_2 + (1+\lambda)^n a_2 u_1(t) w_1 + (1+2\lambda)^n a_3.$$

This implies that

$$\begin{aligned} a_3 = & \frac{u_2(t) w_1^2 + u_1(t) w_2 + (1+\lambda)^n a_2 u_1(t) w_1}{(1+2\lambda)^n \left[(1+2\lambda)^m - 1\right]} \\ = & \frac{u_2(t) w_1^2 + u_1(t) w_2 + \frac{u_1^2(t) w_1^2}{\left[(1+\lambda)^m - 1\right]}}{(1+2\lambda)^n \left[(1+2\lambda)^m - 1\right]}. \end{aligned}$$

Then,

$$\begin{aligned}|a_3| \leq & \frac{1}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\left\{\left(4t^2-1\right)+2t+\frac{(2t)^2}{\left[(1+\lambda)^m-1\right]}\right\}\\ \leq & \frac{1}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\left\{\frac{4(1+\lambda)^m}{\left[(1+\lambda)^m-1\right]}t^2+2t-1\right\}.\end{aligned}$$

□

Theorem 2.2. *Let $f \in A$ of the form (1.1) belong to the class $A(H,n,m,\lambda)$. Then*

$$\left|a_3-\xi a_2^2\right| \leq \begin{cases} \frac{2t}{A}; & \zeta \in [\zeta_1,\zeta_2] \\ \frac{2t}{A}\left|\frac{4t^2-1}{2t}+\frac{2t}{E}-\frac{2tA}{B}\xi\right|; & \zeta \notin [\zeta_1,\zeta_2] \end{cases}$$

where

$$A=(1+2\lambda)^n\left[(1+2\lambda)^m-1\right], \qquad B=(1+\lambda)^{2n}\left[(1+\lambda)^m-1\right]^2, E=(1+\lambda)^m-1,$$

$$\frac{B}{2tA}\left\{\frac{\left(4t^2-2t-1\right)E+4t^2}{2tE}\right\}=\zeta_1$$

and

$$\frac{B}{2tA}\left\{\frac{\left(4t^2+2t-1\right)E+4t^2}{2tE}\right\}=\zeta_2.$$

Proof. From (2.7) and (2.8), we write

$$\begin{aligned}a_3-\xi a_2^2 = & \frac{u_1(t)}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\left\{w_2+\left[\frac{u_2(t)}{u_1(t)}+\frac{u_1^2(t)}{\left[(1+\lambda)^m-1\right]}\right]w_1^2\right\}\\ & -\xi\left\{\frac{u_1(t)w_1}{(1+\lambda)^n\left[(1+\lambda)^m-1\right]}\right]^2\\ = & \frac{u_1(t)}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\left\{w_2+\frac{u_2(t)}{u_1(t)}w_1^2+u_1(t)w_1^2\left[\frac{1}{\left[(1+\lambda)^m-1\right]}\right]\right\}\\ & -\xi\left\{\frac{u_1^2(t)\,w_1^2}{(1+\lambda)^{2n}\left[(1+\lambda)^m-1\right]^2}\right\}\\ = & \frac{u_1(t)}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\\ & \left\{w_2+\frac{u_2(t)}{u_1(t)}w_1^2-u_1(t)\,w_1^2\left[\frac{\xi(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}{(1+\lambda)^{2n}\left[(1+\lambda)^m-1\right]^2}-\frac{1}{\left[(1+\lambda)^m-1\right]}\right]\right\}.\end{aligned} \tag{2.10}$$

Hence, if the absolute value of both sides of equation (2.10) ,we obtain

$$\begin{aligned}\left|a_3-\xi a_2^2\right| = & \frac{2t}{(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}\\ & \left|w_2-\left\{2t\left[\frac{\xi(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]}{(1+\lambda)^{2n}\left[(1+\lambda)^m-1\right]^2}-\frac{1}{\left[(1+\lambda)^m-1\right]}\right]-\frac{4t^2-1}{2t}\right\}w_1^2\right|.\end{aligned} \tag{2.11}$$

Let

$$(1+2\lambda)^n\left[(1+2\lambda)^m-1\right]=A,\quad (1+\lambda)^{2n}\left[(1+\lambda)^m-1\right]^2=B$$

and $(1+\lambda)^m-1=E$. With these abbreviations, the expression (2.11) can be written as

$$\left|a_3-\xi a_2^2\right|=\frac{2t}{A}\left|w_2-\left\{2t\left[\frac{\xi\cdot A}{B}-\frac{1}{E}\right]-\frac{4t^2-1}{2t}\right\}w_1^2\right|. \tag{2.12}$$

Then, in view of (2.6), we conclude that

$$\left|a_3-\xi a_2^2\right|\le\frac{2t}{A}\max\left\{1,\left|2t\left[\frac{\xi\cdot A}{B}-\frac{1}{E}\right]-\frac{4t^2-1}{2t}\right|\right\}.$$

Finally, we obtain

$$\left|2t\left[\frac{\xi\cdot A}{B}-\frac{1}{E}\right]-\frac{4t^2-1}{2t}\right|\le 1$$

if and only if

$$\left|\frac{4t^2-1}{2t}+\frac{2t}{E}-\frac{2tA}{B}\xi\right|\le 1$$

if and only if

$$-1\le\frac{4t^2-1}{2t}+\frac{2t}{E}-\frac{2tA}{B}\xi\le 1$$

if and only if

$$-1-\frac{4t^2-1}{2t}-\frac{2t}{E}\le-\frac{2tA}{B}\xi\le 1-\frac{4t^2-1}{2t}-\frac{2t}{E}$$

if and only if

$$\frac{B}{2tA}\left\{\frac{4t^2-1}{2t}+\frac{2t}{E}-1\right\}\le\xi\le\frac{B}{2tA}\left\{\frac{4t^2-1}{2t}+\frac{2t}{E}+1\right\}$$

if and only if

$$\frac{B}{2tA}\left\{\frac{(4t^2-2t-1)E+4t^2}{2tE}\right\}\le\xi\le\frac{B}{2tA}\left\{\frac{(4t^2+2t-1)E+4t^2}{2tE}\right\}$$

if and only if

$$\zeta_1\le\xi\le\zeta_2.$$

□

Taking $\lambda=1, n=1, m=1$ in the above theorem we get the following result.

Corollary 2.3. *Let $f\in A$ of the form (1.1) belong to the class $A(H,n,m,\lambda)=A(H,1,1,1)$. Then*

$$|a_2|\le t,\qquad |a_3|\le\frac{4}{3}t^2+\frac{t}{3}-\frac{1}{6},$$

and

$$\left|a_3 - \xi a_2^2\right| \leq \begin{cases} \frac{2t}{3}; & \zeta \in [\zeta_1, \zeta_2] \\ \left|\frac{8t^2-1-6\zeta t^2}{6}\right|; & \zeta \notin [\zeta_1, \zeta_2] \end{cases}$$

where $\zeta_1 = \frac{8t^2-2t-1}{12t^2}$ *and* $\zeta_2 = \frac{8t^2+2t-1}{12t^2}$.

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