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Exploring integral-type theorems through fixed-point iteration with C-class functions

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Abstract. In this paper, the concept of C-class functions is used to establish the existence and uniqueness of a fixed point for a self-map of integral type within a complete metric space. The theoretical foundation is presented and the results are exemplified through a specific case. Lastly, an illustration is given to demonstrate the validity of our results.

1. Introduction and Preliminaries

In 2002, Branciari [8] generalized the Banach's contraction principle [6] by introducing a new concept known as the integral type of contraction, and many researchers have studied several fixed point theories for this type of contraction. For example, we refer readers to references ([1], [2], [9], [10], [11], [12], [15], [19]).

The foundational information necessary for elucidating our subsequent results is systematically compiled in this section. By assembling key elements and background details, we lay the groundwork for a comprehensive presentation that enhances the clarity and understanding of our findings.

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Theorem 1.1. [8] *Let (X,d) be a complete metric space and $U : X \to X$ be a self mapping satisfying the contraction*

$$\int_0^{d(Ux,Uy)} \varphi(t)dt \leq \alpha \int_0^{d(x,y)} \varphi(t)dt,$$

where $\alpha \in (0,1)$ and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ which is non-negative, summable on each compact subset of $\mathbb{R}^+$, and such that for each $\varepsilon > 0$, $\int_0^{\varepsilon} \varphi(t)\, dt > 0$. Then U has a unique fixed point in X.

Example 1.1. [8] Let d be a Euclidean distance function, we define the map $U : \mathbb{R}^+ \to \mathbb{R}^+$ and Lebesgue-integrable function φ as

$$U(x) = x + 1 \text{ and } \varphi(t) = -1.$$

Clearly, for some arbitrary $\alpha \in (0,1)$, all the assumptions of Theorem 1.1 are satisfied. But the map U has no fixed point.

Definition 1.2. [16] A function $\psi : [0,\infty) \to [0,\infty)$ is a called an altering distance function if the following properties are satisfied:
1) ψ is non-decreasing and continuous;
2) $\psi(t) = 0$ if and only if $t = 0$.

The concept of C-class functions, introduced by Ansari [3], represents a noteworthy generalization of the traditional Banach contraction. Ansari's innovation extends the scope of fixed-point theorems, offering a more flexible and inclusive framework. Following this, the concept of C-class functions received a great deal of attention from various researchers, resulting in a collective interest in constructing fixed-point theorems in this framework (see [5], [7], [13], [14]).

Definition 1.3. [3] A continuous function $F : [0,+\infty) \times [0,+\infty) \to \mathbb{R}$ is called C-class function if it satisfies the following conditions:
(C1) $F(s,t) \leq s$, for all $(s,t) \in [0,+\infty) \times [0,+\infty)$;
(C2) $F(s,t) = s$ implies that either $s = 0$ or $t = 0$.
Let us suppose that C denote the family of C-class function.

Remark 1.4. [3] Note that for some F, we have that $F(0,0) = 0$.

Example 1.5. [3] The following functions $F : [0,+\infty) \times [0,+\infty) \to \mathbb{R}$ are elements of C, for all $s,t \in [0,+\infty)$:
(1) $F(s,t) = s - t$;
(2) $F(s,t) = ms$, $m \in (0,1)$;
(3) $F(s,t) = \frac{s}{(1+t)^r}$, $r \in [0,+\infty)$;
(4) $F(s,t) = \frac{\log(t+a^s)}{1+t}$, $a > 1$;
(5) $F(s,t) = \frac{\ln(t+a^s)}{2}$, $a > e$;
(6) $F(s,t) = s - \varphi(s)$, where $\varphi : [0,+\infty) \to [0,+\infty)$ is a continuous function such that $\varphi(s) = 0 \Leftrightarrow s = 0$.

In light of the foundational groundwork and the exploration of fixed-point theorems for C-class functions, we now present the main results of our study, unveiling novel insights into the existence and uniqueness of fixed points for self-maps of integral type.

2. Mains results

Theorem 2.1. *Let U be a self mapping on a complete metric space (X,d) satisfying the contraction*

$$\psi\left(\int_0^{d(Ux,Uy)}\varphi(t)\,dt\right)\leq F\left(\psi\left(\int_0^{M(x,y)}\varphi(t)\,dt\right),\delta\left(\int_0^{M(x,y)}\varphi(t)\,dt\right)\right),\qquad(1)$$

for all $x,y\in X$, where $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ which is nonnegative on each compact subset of $\mathbb{R}^+$, such that for each $\epsilon>0$, $\int_0^\epsilon\varphi(t)\,dt>0$ and $F:[0,+\infty[^2\to\mathbb{R}$ is a C-class function, $\psi:[0,+\infty[\to[0,+\infty[$ is an altering distance function and $\delta:[0,+\infty[\to[0,+\infty[$ is a nondecreasing and lower semi continuous such that $\delta(0)\geq 0$, and $\delta(t)>0$, $\forall t>0$, and

$$M(x,y)=k\max\{d(x,y),d(x,Ux),d(y,Uy),d(x,Uy),d(y,Ux)\},\qquad(2)$$

such that $k\in(0,1)$. Then U has a unique fixed point in X.

Proof. Let $x_0\in X$ be arbitrary. Define the sequence $\{x_n\}$ by $x_{n+1}=Ux_n$. For each $n\geq 1$ and from (1), we have

$$\begin{aligned}\psi\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right)&=\psi\left(\int_0^{d(Ux_n,Ux_{n-1})}\varphi(t)\,dt\right)\\&\leq F\left(\psi\left(\int_0^{M(x_n,x_{n-1})}\varphi(t)\,dt\right),\delta\left(\int_0^{M(x_n,x_{n-1})}\varphi(t)\,dt\right)\right),\end{aligned}$$

where

$$\begin{aligned}M(x_n,x_{n-1})&=k\max\left\{\begin{array}{c}d(x_n,x_{n-1}),d(x_n,Ux_n),d(x_{n-1},Ux_{n-1}),\\ d(x_n,Ux_{n-1}),d(x_{n-1},Ux_n)\end{array}\right\}\\&=k\max\left\{\begin{array}{c}d(x_n,x_{n-1}),d(x_n,x_{n+1}),d(x_{n-1},x_n),\\ d(x_n,x_n),d(x_{n-1},x_{n+1})\end{array}\right\}\\&=k\max\{d(x_n,x_{n-1}),d(x_n,x_{n+1}),d(x_{n-1},x_{n+1})\}\\&=k\max\{d(x_n,x_{n-1}),d(x_n,x_{n+1})\}.\end{aligned}$$

First, we suppose that $d(x_n,x_{n+1})\geq d(x_n,x_{n-1})$ for some n, we get

$$\psi\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right)\leq F\left(\psi\left(\int_0^{kd(x_{n+1},x_n)}\varphi(t)\,dt\right),\delta\left(\int_0^{kd(x_{n+1},x_n)}\varphi(t)\,dt\right)\right).$$

Now, we conclude immediately that

$$\psi\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right) \le \psi\left(\int_0^{kd(x_{n+1},x_n)}\varphi(t)\,dt\right),$$

which contradicts. Hence $M(x_n,x_{n-1}) = kd(x_n,x_{n-1})$, so

$$\psi\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right) \le F\left(\psi\left(\int_0^{kd(x_n,x_{n-1})}\varphi(t)\,dt\right),\delta\left(\int_0^{kd(x_n,x_{n-1})}\varphi(t)\,dt\right)\right),$$

which conclude that

$$\psi\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right) \le \psi\left(\int_0^{kd(x_n,x_{n-1})}\varphi(t)\,dt\right).$$

Now, we will show that $\{x_n\}$ is a Cauchy sequence in (X,d), for some $n \ge 1$, we have

$$\begin{aligned}\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt &\le \int_0^{kd(x_n,x_{n-1})}\varphi(t)\,dt\\ &\le \int_0^{k^2d(x_{n-1},x_{n-2})}\varphi(t)\,dt\\ &\;\;\vdots\\ &\le \int_0^{k^nd(x_1,x_0)}\varphi(t)\,dt.\end{aligned}$$

Taking $n \to \infty$, we get

$$\lim_{n\to\infty}\left(\int_0^{d(x_{n+1},x_n)}\varphi(t)\,dt\right) = 0.$$

Consequently, it gives

$$\lim_{n\to\infty} d(x_{n+1},x_n) = 0. \tag{3}$$

Next, we assert that the sequence $\{x_n\}$ is Cauchy. Assume not, so for $\varepsilon > 0$, there exists subsequences $\{x_{n(i)}\}$ and $\{x_{m(i)}\}$ of $\{x_n\}$ with $n(i) < m(i) < n(i+1)$ satisfying

$$d\left(x_{n(i)},x_{m(i)}\right) \ge \varepsilon \text{ and } d\left(x_{n(i)},x_{m(i-1)}\right) < \varepsilon. \tag{4}$$

Consider

$$\begin{aligned}
&\psi\left(\int_0^{\varepsilon}\varphi(t)\,dt\right)\leq\psi\left(\int_0^{d\left(x_{n(i)},x_{m(i)}\right)}\varphi(t)\,dt\right)\\
=&\psi\left(\int_0^{d\left(Ux_{n(i)-1},Ux_{m(i)-1}\right)}\varphi(t)\,dt\right)\\
\leq&F\left(\psi\left(\int_0^{M\left(x_{n(i)-1},x_{m(i)-1}\right)}\varphi(t)\,dt\right),\delta\left(\int_0^{M\left(x_{n(i)-1},x_{m(i)-1}\right)}\varphi(t)\,dt\right)\right),
\end{aligned}\tag{5}$$

from (2), we find

$$\begin{aligned}
&M\left(x_{n(i)-1},x_{m(i)-1}\right)\\
=&k\max\left\{d\left(x_{n(i)-1},x_{m(i)-1}\right),d\left(x_{n(i)-1},Ux_{n(i)-1}\right),d\left(x_{m(i)-1},Ux_{m(i)-1}\right),\right.\\
&\left.d\left(x_{n(i)-1},Ux_{m(i)-1}\right),d\left(x_{m(i)-1},Ux_{n(i)-1}\right)\right\}\\
=&k\max\left\{d\left(x_{n(i)-1},x_{m(i)-1}\right),d\left(x_{n(i)-1},x_{n(i)}\right),d\left(x_{m(i)-1},x_{m(i)}\right),\right.\\
&\left.d\left(x_{n(i)-1},x_{m(i)}\right),d\left(x_{m(i)-1},x_{n(i)}\right)\right\},
\end{aligned}$$

where

$$\begin{aligned}
&\int_0^{M\left(x_{n(i)-1},x_{m(i)-1}\right)}\varphi(t)\,dt\\
=&\int_0^{k\max\left\{d\left(x_{n(i)-1},x_{m(i)-1}\right),d\left(x_{n(i)-1},x_{n(i)}\right),d\left(x_{m(i)-1},x_{m(i)}\right),d\left(x_{n(i)-1},x_{m(i)}\right),d\left(x_{m(i)-1},x_{n(i)}\right)\right\}}\varphi(t)\,dt\\
=&\max\left\{\int_0^{kd\left(x_{n(i)-1},x_{m(i)-1}\right)}\varphi(t)\,dt,\int_0^{kd\left(x_{n(i)-1},x_{n(i)}\right)}\varphi(t)\,dt,\int_0^{kd\left(x_{m(i)-1},x_{m(i)}\right)}\varphi(t)\,dt,\right.\\
&\left.\int_0^{kd\left(x_{n(i)-1},x_{m(i)}\right)}\varphi(t)\,dt,\int_0^{kd\left(x_{m(i)-1},x_{n(i)}\right)}\varphi(t)\,dt\right\}.
\end{aligned}$$

By using (4) and triangle inequality, we find

$$\begin{aligned}
kd(x_{n(i)-1},x_{m(i)-1})&\leq kd(x_{n(i)-1},x_{n(i)})+kd(x_{n(i)},x_{m(i)-1})\\
&\leq kd(x_{n(i)-1},x_{n(i)})+k\varepsilon,
\end{aligned}$$

$$\lim_{i\to\infty}\int_0^{kd(x_{n(i)-1},x_{m(i)-1})}\varphi(t)\,dt\leq\int_0^{k\varepsilon}\varphi(t)\,dt,\tag{6}$$

and

$$\begin{aligned}
kd\left(x_{n(i)-1},x_{m(i)}\right)&\leq kd\left(x_{n(i)-1},x_{n(i)}\right)+kd\left(x_{n(i)},x_{m(i)}\right)\\
&\leq kd\left(x_{n(i)-1},x_{n(i)}\right)+kd\left(x_{n(i)},x_{m(i)-1}\right)+kd\left(x_{m(i)-1},x_{m(i)}\right)\\
&\leq kd\left(x_{n(i)-1},x_{n(i)}\right)+k\varepsilon+kd\left(x_{m(i)-1},x_{m(i)}\right),
\end{aligned}$$

taking i to infinity and using (3), we have

$$\lim_{i\to\infty}\int_0^{kd\left(x_{n(i)-1},x_{m(i)}\right)}\varphi(t)\,dt\leq\int_0^{k\varepsilon}\varphi(t)\,dt, \tag{7}$$

form (4), we get

$$\int_0^{kd\left(x_{m(i)-1},x_{n(i)}\right)}\varphi(t)\,dt\leq\int_0^{k\varepsilon}\varphi(t)\,dt. \tag{8}$$

Taking i to infinity in inequality (5) and using (6),(7),(8), we get

$$\psi\left(\int_0^{\varepsilon}\varphi(t)\,dt\right)\ \leq\ F\left(\psi\left(\int_0^{k\varepsilon}\varphi(t)\,dt\right),\delta\left(\int_0^{k\varepsilon}\varphi(t)\,dt\right)\right).$$

So,

$$\psi\left(\int_0^{\varepsilon}\varphi(t)\,dt\right)\ \leq\ \psi\left(\int_0^{k\varepsilon}\varphi(t)\,dt\right)$$

this is a contradiction. Therefore $\{x_n\}$ is a Cauchy sequence. Call the limit as x such that $\lim\limits_{n\to\infty}x_n=x$, i.e

$$\lim_{n\to\infty}Ux_n=x,$$

Next, assert that x is a fixed point of U, consider

$$\lim_{n\to\infty}\psi\left(\int_0^{d(Ux_n,Ux)}\varphi(t)\,dt\right)\leq\lim_{n\to\infty}F\left(\psi\left(\int_0^{M(x_n,x)}\varphi(t)\,dt\right),\delta\left(\int_0^{M(x_n,x)}\varphi(t)\,dt\right)\right) \tag{9}$$

where

$$\lim_{n\to\infty}M(x_n,x)=\lim_{n\to\infty}k\max\left\{d(x_n,x),d(x_n,Ux_n),d(x,Ux),d(x_n,Ux),d(x,Ux_n)\right\}.$$

Then

$$\lim_{n\to\infty}M(x_n,x)=kd(x,Ux).$$

From (9), we get

$$\begin{aligned}\psi\left(\int_0^{d(x,Ux)}\varphi(t)\,dt\right)\ &\leq\ F\left(\psi\left(\int_0^{kd(x,Ux)}\varphi(t)\,dt\right),\delta\left(\int_0^{kd(x,Ux)}\varphi(t)\,dt\right)\right)\\ &\leq\ \psi\left(\int_0^{kd(x,Ux)}\varphi(t)\,dt\right),\end{aligned}$$

this is a contradiction. This implies $d(x,Ux)=0$. Thus $Ux=x$. This proves that x is a fixed point of U. For the uniqueness, assume that there is another point y such that $Uy=y$, from (1), we have

$$\psi\left(\int_0^{d(Uy,Ux)}\varphi(t)\,dt\right)\leq F\left(\psi\left(\int_0^{M(y,x)}\varphi(t)\,dt\right),\delta\left(\int_0^{M(y,x)}\varphi(t)\,dt\right)\right),$$

where

$$\begin{aligned} M(y,x) &= k\max\{d(x,y), d(x,Ux), d(y,Uy), d(x,Uy), d(y,Ux)\} \\ &= kd(x,y). \end{aligned}$$

Thus

$$\begin{aligned} \psi\left(\int_0^{d(Uy,Ux)} \varphi(t)\,dt\right) &= \psi\left(\int_0^{d(y,x)} \varphi(t)\,dt\right) \\ &\le F\left(\psi\left(\int_0^{kd(x,y)} \varphi(t)\,dt\right), \delta\left(\int_0^{kd(x,y)} \varphi(t)\,dt\right)\right). \end{aligned}$$

Thus

$$\psi\left(\int_0^{d(Uy,Ux)} \varphi(t)\,dt\right) \le \psi\left(\int_0^{kd(x,y)} \varphi(t)\,dt\right).$$

This is a contradiction. Therefore $x = y$. Hence x is the unique fixed point of U. □

As a direct consequence of our main results, we derive the following corollaries

Corollary 2.2. *Let U be a self mapping on a complete metric space (X,d), satisfying the contraction*

$$\psi\left(\int_0^{d(Ux,Uy)} \varphi(t)\,dt\right) \le \alpha\psi\left(\int_0^{M(x,y)} \varphi(t)\,dt\right),$$

for $0 < \alpha < 1$, for all $x, y \in X$, where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ which is nonnegative on each compact subset of $\mathbb{R}^+$, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)\,dt > 0$, and $\psi : [0,+\infty[\to [0,+\infty[$ is an altering distance function, and

$$M(x,y) = k\max\{d(x,y), d(x,Ux), d(y,Uy), d(x,Uy), d(y,Ux)\}$$

such that $k \in (0,1)$. Then U has a unique fixed point in X.

Proof. Put $F(s,t) = ms$, where $m \in (0,1)$ in Theorem 2.1, we get the result □

Corollary 2.3. *Let U be a self mapping on a complete metric space (X,d), satisfying the contraction*

$$\psi\left(\int_0^{d(Ux,Uy)} \varphi(t)\,dt\right) \le \psi\left(\int_0^{M(x,y)} \varphi(t)\,dt\right) - \delta\left(\int_0^{M(x,y)} \varphi(t)\,dt\right),$$

for all $x, y \in X$, where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ which is nonnegative on each compact subset of $\mathbb{R}^+$, and such that for each $\epsilon > 0$, $\int_0^{\epsilon} \varphi(t)\,dt > 0$, and $\psi : [0,+\infty[\to [0,+\infty[$ is an altering distance function and $\delta : [0,+\infty[\to [0,+\infty[$ is a nondecreasing and lower semi continuous such that $\delta(0) \ge 0$, and $\delta(t) > 0$, $\forall t > 0$, and $M(x,y) =$

$k\max\left\{d(x,y),d(x,Ux),d\left(y,Uy\right),d\left(x,Uy\right),d\left(y,Ux\right)\right\}$, *such that* $k\in\left(0,1\right)$.
Then U *has a unique fixed point in* X.

Proof. Put $F(s,t)=s-t$, in Theorem 2.1, we get the result. □

Corollary 2.4. *Let* U *be a self mapping on a complete metric space* (X,d), *satisfying the contraction*

$$\psi\left(\int_0^{d(Uy,Ux)}\varphi\left(t\right)dt\right)\leq\frac{\psi\left(\int_0^{M(y,x)}\varphi\left(t\right)dt\right)}{1+\delta\left(\int_0^{M(y,x)}\varphi\left(t\right)dt\right)},$$

for all $x,y\in X$, *where* $\varphi:\mathbb{R}^+\to\mathbb{R}^+$ *which is nonnegative on each compact subset of* $\mathbb{R}^+$, *and such that for each* $\epsilon>0$, $\int_0^{\epsilon}\varphi\left(t\right)dt>0$,*and* $\psi:[0,+\infty[\to[0,+\infty[$ *is an altering distance function and* $\delta:[0,+\infty[\to[0,+\infty[$ *is a nondecreasing and lower semi continuous such that* $\delta\left(0\right)\geq0$, *and* $\delta\left(t\right)>0$, $\forall t>0$, *and* $M(x,y)=k\max\left\{d(x,y),d(x,Ux),d\left(y,Uy\right),d\left(x,Uy\right),d\left(y,Ux\right)\right\}$, *such that* $k\in\left(0,1\right)$.
Then U *has a unique fixed point in* X.

Proof. Put $F\left(s,t\right)=\frac{s}{\left(1+t\right)^r}$, and assume $r=1$ in Theorem 2.1, we obtain the result. □

To illustrate the practical implications of our main results, we present a compelling example that demonstrates the application of the derived fixed-point theorems for C-class functions in a specific mathematical context. The theoretical findings are concretized through this example.

Example 2.1. Let $X=[0,1]$ and $d\left(x,y\right)=\left|x-y\right|$. We take $U:X\to X$ by

$$Ux=\frac{1}{2}x.$$

Define $F:\left[0,+\infty\right)^2\to\mathbb{R}$ as

$$F\left(s,t\right)=ms\text{ with }m=\frac{3}{4}\in\left(0,1\right),$$

then F is a C-class function (from Example 1.5). Let as define $\psi,\varphi:[0,+\infty)\to[0,+\infty)$ by $\psi\left(t\right)=t$ and $\varphi\left(t\right)=\frac{t}{3}$. Then, for each $\varepsilon>0$,

$$\int_0^{\varepsilon}\varphi\left(t\right)dt=\frac{\varepsilon^2}{6}>0$$

and

$$M(x,y)=k\max\left\{d(x,y),d(x,Ux),d\left(y;Uy\right),d\left(x,Uy\right),d\left(y,Ux\right)\right\}$$

such that

$$0<k=\frac{1}{\sqrt{2}}<1.$$

We can verify the contraction of Theorem 2.1. If $x = y$ for all $x, y \in X$, then result holds trivially. So suppose that $x \neq y$ for all $x, y \in X$, we get

$$d(x,y) = 1,\ d(x,Ux) = 0,\ d(y,Uy) = \frac{1}{2},\ d(x,Uy) = \frac{1}{2},\ d(y,Ux) = 1,$$

we obtain

$$\begin{aligned} M(x,y) &= k \max\{d(x,y), d(x,Ux), d(y,Uy), d(x,Uy), d(y,Ux)\} \\ &= \frac{1}{\sqrt{2}} \end{aligned}$$

and

$$\begin{aligned} \psi\left(\int_0^{d(Ux,Uy)} \varphi(t)\, dt\right) &= \int_0^{|Ux-Uy|} \frac{t}{3} dt \\ &= \frac{1}{24}. \end{aligned}$$

So

$$\begin{aligned} F\left(\psi\left(\int_0^{M(x,y)} \varphi(t)\, dt\right), \delta\left(\int_0^{M(x,y)} \varphi(t)\, dt\right)\right) &= m\psi\left(\int_0^{M(x,y)} \varphi(t)\, dt\right) \\ &= m\int_0^{M(x,y)} \varphi(t)\, dt = \frac{1}{16}. \end{aligned}$$

Then clearly

$$\psi\left(\int_0^{d(Ux,Uy)} \varphi(t)\, dt\right) \leq F\left(\psi\left(\int_0^{M(x,y)} \varphi(t)\, dt\right), \delta\left(\int_0^{M(x,y)} \varphi(t)\, dt\right)\right).$$

Hence, all the conditions of the Theorem 2.1 are verified and 0 is the unique fixed point of the map U.

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