Mathematical Analysis and its Contemporary Applications
Volume 6, Issue 2, 2024, 1–15
doi: 10.30495/maca.2024.2028255.1099
ISSN 2716-9898

Some remarks on generalized Sehgal-Guseman-Like contractions and their fixed-point results

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Abstract. The aim of this paper is to shed some light on the recently introduced generalized Sehgal-Guseman-like contractions with the help of D-functions from other aspects. For this purpose, we used the recently established connection of S-metric and b-metric spaces and thus switched to a new form of generalized Sehgal-Guseman-like contraction. Using that connection as well as the notion of a D-function, we were able to significantly improve recent results on generalized Sehgal-Guseman-like contractions and remove some of the confusion readers had when reading recent work on this topic. At the end of the paper, we listed all 120 possible generalized Sehgal-Guseman-like contractions within both S-metric and b-metric spaces. We admit that, for many of them, we do not yet know whether they have a (unique) fixed point. This opens the way for further research of generalized Sehgal-Guseman-like contractions in the mentioned frameworks of certain generalized metric spaces.

1. Introduction and Preliminaries

Wanting to generalize Banach's result from 1922, many authors broke the metric space axiom and came up with new types of space where distance is measured in a

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2020 *Mathematics Subject Classification.* Primary 47H10; Secondary 54H25.

Key words and phrases. S-metric space, b-metric space, fixed point, generalized Sehgal-Guseman-like-contraction, $\mathcal{D}-$class functions.

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different way than the classical Fréchet metric. One of those types of spaces are the S-metric spaces introduced in 2012 by S. Sedghi et al., [26].

We state their definition and basic properties such as convergence, Cauchyness, completeness and mapping continuity. In 1969 and 1970, two significant results were obtained that look like generalizations of Banach's well-known theorem from 1922. These are the following two theorems:

Theorem 1.1. [25] *Let* (X, d) *be a complete metric space, and* $f : X \to X$ *a continuous mapping satisfying the condition: there exists a* $k < 1$ *such that for each* $x \in X$*, there is a positive integer* $n(x)$ *such that for all* $y \in X$

$$d(f^{n(x)}(y), f^{n(x)}(x)) \leq k \cdot d(y, x).$$

Then f *has a unique fixed point* u *and* $f^n(x_0) \to u$ *when* $n \to +\infty$*, for each* $x_0 \in X$*.*

Theorem 1.2. [12] *Let* (X, d) *be a complete metric space and let* $f : X \to X$ *be a mapping. Suppose there exists* $B \subset X$ *such that*

(a) $f(B) \subset B$,

(b) for some $k < 1$ and each $y \in B$ there is an integer $n(y) \geq 1$ with $d\left(f^{n(y)}(x), f^{n(y)}(y)\right) \leq k \cdot d(x, y)$ for all $x \in B$, and

(c) for some $x_0 \in B, cl\{f^n(x_0) : n \geq 1\} \subset B$.

Then there is a unique $u \in B$ such that $f(u) = u$ and $f^n(y_0) \to u$ when $n \to +\infty$, for each $y_0 \in B$. Furthermore, if $d\left(f^{n(u)}(x), f^{n(u)}(u)\right) \leq k \cdot d(x, u)$ for all $x \in X$, then u is unique in X and $f^n(x_0) \to u$ when $n \to +\infty$, for each $x_0 \in X$.

Recently, M. Din and all., [6], tried to introduce Sehgal-Guseman-like contractions within S-metric spaces and prove the corresponding theorems. The aim of this paper is to critically reflect on their attempt. In it, we will use the usual marks that are traditionally used, which will improve its readability, and therefore a better understanding of the entire essence of the matter that will be presented in the paper. Which is not the case with the work we are analyzing here. We begin with definitions of basic terms in the class of S-metric spaces.

Definition 1.3. [26] Let X be a non-empty set and denote by S the mapping from X^3 to $\mathbb{R}_+$ that satisfies the following axioms:

(S1): $S(x, y, z) = 0$ if and only if $x = y = z$;

(S2): $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ for all x, y, z, a from X.

Then the pair (X, S) is called an S-metric space and the mapping S is called a S-metric on X.

If instead (S2) we take the following axioms:

(S2′): $S(x, y, z) \leq s \cdot [S(x, x, a) + S(y, y, a) + S(z, z, a)]$ for all x, y, z, a from X where $s \geq 1$, then we say that $(X, S, s \geq 1)$ is a S_b-metric space.

Some examples of S-metric spaces:

Example 1. Let $\|.\|$ a norm on the vector space V, then $S(x,y,z) = \|y+z-2x\| + \|y-z\|$ is an $S-$metric on V.

Example 2. Let (X,d) be a metric space, then $S(x,y,z) = d(x,z) + d(y,z)$ is an $S-$metric on the set X.

Properties such as convergence of a sequence, Cauchyness of a sequence, completeness of the space and continuity of a function, all within S-metric spaces are given by the following definition:

Definition 1.4. [26] Let (X,S) be an S-metric space.

(1) A sequence $\{x_n\}$ in X converges to x if and only if $S(x_n,x_n,x) \to 0$ as $n \to +\infty$.

(2) A sequence $\{x_n\}$ in X is called a Cauchy sequence if $S(x_n,x_n,x_m) \to 0$ as $n,m \to +\infty$.

(3) The S-metric space (X,S) is said to be complete if every Cauchy sequence is convergent.

(4) A mapping $T : X \to X$ is said to be $S-$continuous if $\{Tx_n\}$ is $S-$convergent to Tx, where $\{x_n\}$ is an $S-$convergent sequence converging to x.

For further details reader can see the following papers: [1], [3], [5]-[9], [11], [23], [24], [26]-[29], [31].

Similarly to metric and G-metric spaces, open and closed balls are defined and the corresponding topology is based on them. For details, see the papers on S-metric spaces in the reference list. Here, the sequence converges on the S-metric if and only if it converges on that resulting topology. It is well known that such equivalence does not hold for b-metric, G_b-metric and S_b-metric spaces. This is because the open sphere defined in them does not have to be open in the generated topology.

Using the connection of S-metric and b-metric spaces that is given and explained in the following Proposition, we will provide in this paper a substantial correction of the result from [6]. Now we state the position on the relationship between S-metric and b-metric spaces:

Proposition 1.5. *Let (X,S) be an S-metric space. Then with $b(x,y) = S(x,x,y)$ one b-metric on the set X is given.*

The following applies:

a) (X,S) is complete S-metric space if and only if (X,b) is a complete b-metric space;

b) Sequence x_n converges in S-metric space (X,S) if and only if it converges in b-metric space (X,b);

c) The same applies when the sequence x_n is a Cauchy. Namely, it is Cauchy in (X,S) if and only if it is Cauchy in the b-metric space (X,b);

d) The mapping T from X to itself is continuous in (X,S) if and only if it is continuous in (X,b);

e) Since S is a continuous function with three variables, then the newly defined b-metric b is also such, i.e., a continuous function with two variables.

Remark 1.6. For a proof of the mentioned properties, see the recent interesting paper [**27**]. Thus, the coefficient s in the obtained b-metric space is equal to $\frac{3}{2}$. Let us also mention one error from the work of G. S. Saluje [**24**]: If (X,S) is a given S-metric space, then with $d_G(x,y)=S(x,x,y)+S(y,y,x)$ there is one metric on the set X. According to Proposition 3., it is false. Indeed, if the above equality were possible, then we would have that the metric d_G is equal to the $2\cdot b$ from Proposition 1. But since the coefficient s of the b-metric b is equal to $\frac{3}{2}>1$, it means that b is not a metric. It follows from the assertion of the author in [**24**] that $b=\frac{1}{2}d_G$, i.e., that b is a metric, because obviously $\frac{1}{2}d_G$ is a metric.

When different points x and y as well as Tx and Ty are observed in some metric (b-metric) space (X,d), where T is the mapping from X into itself, we get a quadrilateral with vertices x,y,Tx,Ty. There are also lengths determined by points x,Ty and y,Tx, i.e., diagonals. Then we have 6 distances. If, for example, we measure-estimate $d(Tx,Ty)$ over the other distances (of which there are 5), then 5 variables can participate in each contractive condition (if the distances are on the first degree, for example, etc ...). Therefore, for the consideration of a general contractive condition consider the vector space $\mathbb{R}^5$ over the field of real numbers. Two elements in it are comparable by coordinates, i.e., (a,b,c,d,e) is before (a_1,b_1,c_1,d_1,e_1) if and only if is $a\le a_1,b\le b_1,c\le c_1,d\le d_1,e\le e_1$ where "$\le$" is an ordinary relation in the set of real numbers $\mathbb{R}$. Therefore, some contractive condition can be written in the form $d(Tx,Ty)\le g[d(x,y),d(x,Tx),d(y,Ty),d(x,Ty),d(y,Tx)]$ where certain properties are attributed to the mapping g. Note that g maps $\mathbb{R}^5_+$ to the set of positive numbers $\mathbb{R}^+$.

To define a contractive condition within S-metric space corresponding to a given mapping T from an S-metric space to itself and involving some iteration of the mapping T, similar to the Sehgal-Guseman condition within ordinary metric spaces, the authors M.Din et al.[**6**], first introduce a class of mappings from $\mathbb{R}^5_+$ to $\mathbb{R}_+$ called $\mathcal{D}$-functions as follows:

Definition 1.7. Let $\mathcal{D}$ be the set of all continuous mappings g from $\mathbb{R}^5_+$ to $\mathbb{R}_+$ such that the following two properties hold:

1. For each $p,q,r\in\mathbb{R}_+$ if $r\le g[p,2p+r,r,p+2q,q]$ with $q\le 2(p+r)$ then $r\le\phi\cdot p$ for some $\phi\in[0,1)$.
2. If $w\preceq h$, then $g(w)\le g(h)$ for all $w,h\in\mathbb{R}^5_+$.

Here are three of their characteristic examples of class $\mathcal{D}$. ([**6**], page 3).

Example 3. Define $g_1 : \mathbb{R}_+^5 \to \mathbb{R}_+$ such that $g_1(a_1, a_2, a_3, a_4, a_5) = \phi \cdot a_1$ where $\phi \in [0, 1)$. Then, $g_1 \in \mathcal{D}$.

Example 4. Define $g_2 : \mathbb{R}_+^5 \to \mathbb{R}_+$ such that $g_2(a_1, a_2, a_3, a_4, a_5) = \phi a_1 + \psi a_2 + \zeta[a_3 + a_4] + \nabla a_5$ where $\phi + \psi + 3\zeta + \nabla \in [0, 1)$. Then, $g_2 \in \mathcal{D}$.

Example 5. Define $g_3 : \mathbb{R}_+^5 \to \mathbb{R}_+$ such that $g_3(a_1, a_2, a_3, a_4, a_5) = \phi \cdot a_1^{\zeta} \cdot a_2^{\psi} \cdot a_5^{1-\psi-\zeta}$, where $\phi \in [0, 1)$ and $\psi + \zeta \in (0, 1)$. Then, $g_3 \in \mathcal{D}$.

With the following definition they introduce what they say is a generalized Sehgal-Guseman-like contraction:

Definition 1.8. ([6], Definition 4.) Let (X, S) be an S-metric space. A mapping $T : X \to X$ is considered as a generalized Sehgal-Guseman-like contraction if there exists a positive integer m and $g \in \mathcal{D}$ such that

$$\begin{aligned}&S(T^m x, T^m x, T^m y)\\ \leq &g\left[S(x, x, y),\ S(T^m x, T^m x, x), S(T^m x, T^m x, y), S(T^m y, T^m y, x), S(T^m y, T^m y, y)\right]\end{aligned}$$

for all $x, y \in X$ provided that $S(T^m x, T^m x, T^m y) > 0$.

They further formulate the corresponding Theorem for the introduced type of contraction in the framework of S-metric spaces.

Theorem 1.9. *([6], Theorem 2.) Let (X, S) be a complete $S-$metric space and $T : X \to X$ a generalized Sehgal-Guseman-like contraction. Then T possesses a unique fixed point.*

Since in Definition 3 of the paper [6] one needs to "guess" the useful order of the components of the quintet (a, b, d, e, f) in order to arrive at a fixed point of the mapping T of the set X into itself by a valid and usual procedure. A close inspection of the proof of Theorem 2. in [6] shows that the authors did not choose that useful layout. In the corrected arrangement of the components of the quintet, we state the new contractive condition, which in some way improves Definition 4. and Theorem 2. both from [6].

Using Proposition 3. and the b-metric b obtained in it, the contractive condition in Definition 4, i.e., in the formulation of Theorem 4, takes the following form:

$$b(T^m x, T^m y) \leq g\left[b(x, y), b(T^m x, x), b(T^m y, y), b(T^m x, y), b(T^m y, x)\right]$$

for all $x, y \in X$ provided that $b(T^m x, T^m y) > 0$.

Remark 1.10. An important note. Observe that definition 4. does not provide a Sehgal-Guseman type contraction because the definition does not state that for every $x \in X$ there is an integer $n(x)$ so that some condition is met. Therefore, the degree n of the iteration of the mapping T must depend on x from X. This is the main feature of the Sehgal-Guseman contraction definition. If we introduce the shift

$T^m = Q$ in Definition 4., then in Theorem 5. that follows, it should be proved that Q has a unique fixed point using $\mathcal{D}$-functions. It follows directly that T also has a unique fixed point. We will try to show this in the part of the paper entitled **Main results**, which will also corroborate their proof presented in paper [6].

2. Main results

2.1. Improved results. Using the relation of the S-metric S and the b-metric b induced-given by the S-metric S the formulations of Definition 4. and Theorem 2. from the paper [6] now reads:

Definition 2.1. Let (X, S) be a S-metric space and (X, b) be a corresponding b-metric space according to Proposition 3. A mapping $T : X \to X$ is considered as a generalized Sehgal-Guseman-like contraction if there exists a positive integer a and $g \in \mathcal{D}$ such that

$$b\left(T^a x, T^a y\right) \leq g\left[b\left(x, y\right), b\left(T^a x, x\right), b\left(T^a x, y\right), b\left(T^a y, x\right), b\left(T^a y, y\right)\right],$$

for all $x, y \in X$ provided that $b\left(T^a x, T^a y\right) > 0$.

Theorem 2.2. *Let (X, S) be a complete S-metric space and (X, b) be a corresponding b-metric space according to Proposition 3 and $T : X \to X$ a generalized Sehgal-Guseman-type contraction. Then, T possesses a unique fixed point.*

Now we are ready to significantly improve the proof of Theorem 2., and thus bring the notion of Sehgal-Guseman-like contraction within S-metric spaces closer to many readers and especially to young researchers. The famous Definition 3. from [6] as well as points 1. and 2. in it helped us a lot for that.

Our first result in this part of the paper will represent our approach to proving Theorem 2 from [6]. In the very formulation of their Theorem 2. , we will replace the S-metric S with the b-metric b according to Proposition 3., from the introduction of our paper. Our proof will have only two steps and will represent a significant improvement and clarification of the proof given by the authors in [6]. The first step will be to prove the uniqueness of the fixed point if it exists. In the second step, we will analyze the ratio of consecutive points x_{n-1} and x_n of the Picard sequence x_n generated by an arbitrary given point x_0 from X.

First step. Suppose that the mapping $Q = T^m$ has two distinct fixed points u and v. Putting $x = u, y = v$ in the newly obtained contractive condition after replacing the S-metric S with the b-metric b, we get:

$$\begin{aligned} b(u,v) \leq & g[b(u,v), b(u,u), b(u,v), b(u,v), b(v,v)] \\ = & g[b(u,v), 0, b(u,v), b(u,v), 0]. \end{aligned}$$

Comparing the obtained with point 1. from Definition 3., work [6] we have the following: $p = r = b(u,v), 2p + r = 0, p + 2q = b(u,v), q = 0$ It follows that

$b(u,v)=0$, i.e., that $u=v$ which contradicts the assumption that u and v are two different fixed points. We note that the condition $q\leq 2(p+r)$ is then fulfilled as well as the consequence that $r\leq\phi\cdot p$ for some ϕ from $[0,1)$. Thus, we proved that a possible fixed point of the mapping Q is unique.

Second step. Let $x_n=Qx_{n-1}$ be the Picard sequence generated by a given fixed point x_0 of X. If for some n from $\mathbb{N}$ we have that $x_{n-1}=x_n$ then according to **the first step** x_n is a unique fixed point of mapping Q. Suppose then that $x_{n-1}\neq x_n$ for every n in $\mathbb{N}$. Now putting $x=x_{n-1}$, $y=x_n$ in the corresponding previously mentioned contractive condition we get

$$b(x_n,x_{n+1})\leq g[b(x_{n-1},x_n),b(x_{n-1},x_n),b(x_n,x_n),b(x_{n-1},x_{n+1}),b(x_n,x_{n+1})].$$

Comparing, as in the first step obtained with point 1. from Definition 3., paper [6], we have the following: $r=0,p=2p+r$, which gives us that $b(x_{n-1},x_n)=0$. We therefore obtained a contradiction with the assumption that x_{n-1} is different from x_n for every n in $\mathbb{N}$. This further means that there exists n in $\mathbb{N}$ such that $x_{n-1}=x_n$ for some n in $\mathbb{N}$. According to the first step, we obtained that x_n is then a unique fixed point of the mapping Q. According to the well-known fixed point result, this also means that the mapping T has a unique fixed point in the set X. The proof of their Theorem 2. is completely completed. □

Before moving on to presenting the proof of one of our main results of this paper, we mention the Lemma 2.2 from ([17] 2017) by two Romanian mathematicians, which has gained great popularity among researchers in metric fixed point theory. It reads:

Lemma 2.3. *Let* $\{x_n\}$ *be a sequence in a b-metric space* $(X,d,s\geq 1)$*such that* $d(x_n,x_{n+1})\leq\lambda d(x_{n-1},x_n)$ *for some* $\lambda\in[0,1)$*, and each* $n\in\mathbb{N}$*. Then,* $\{x_n\}$ *is a b-Cauchy sequence in* $(X,d,s\geq 1)$*.*

Proof. Theorem 2 from [6] but with a new (more natural) generalized Sehgal-Guseman-like contractive condition given on page 5 near the top.

In this part of the paper, we will give a new proof of Theorem 2. from [6], applying the continuous b-metric which, according to Proposition 3, originates from the given S-metric. Our method is also a correction of their proof, which has several weaknesses (for example, the excess of step 3 as well as the deficiency in the proof of Cauchyness of the Picard sequence - page 5, last line). In the proof, we start from the contractive condition given on page 5, in which the b-metric b participates.

In the **first step**, we show that a possible fixed point of the mapping $Q=T^m$ is unique. For this purpose, let us assume that there are two distinct fixed points u and v of the mapping Q. By replacing x with u and y with v in the given contractive condition, we get

$$b\left(u,v\right)\leq g\left[b\left(u,v\right),b\left(u,u\right),b\left(v,v\right),b\left(v,u\right),b\left(v,u\right)\right]$$

$$=g\left[b\left(u,v\right),0,0,b\left(u,v\right),b\left(u,v\right)\right].$$

From there we get a comparison with point 1. of Definitions 3. that is, $p=b\left(u,v\right), 2p+r=0, r=0, p+2q=b\left(u,v\right)+2b\left(u,v\right), q=b\left(u,v\right)$. We further read and check that $q\leq 2(p+r)$ is true. We now see that because $p=b\left(u,v\right), 2p+r=0, r=0$ it follows that $b(u,v)=0$, i.e., $u=v$ which is a contradiction with the assumption that u and v are different. We also see that the consequence $r\leq\phi\cdot p$ is also true because $0\leq\phi\cdot b(u,v)=\phi\cdot 0=0$.

In the **second step** of the proof, we check whether the Picard sequence $x_n=Qx_{n-1}, n=1,2,\dots$ which is generated by an arbitrary point x_0 from X is a Cauchy. First of all, we notice that if $x_n=x_{n-1}$ for some n, then according to the first step x_{n-1} is a unique fixed point of the mapping Q and then the Theorem is proved. Assume further that x_n is different from x_{n-1} for every n in $\mathbb{N}$. Putting in the given contractive condition $x=x_{n-1}, y=x_n$ we get

$$\begin{aligned}b(x_n,x_{n+1})\leq g&\left[b\left(x_{n-1},x_n\right),b\left(x_n,x_{n-1}\right),b\left(x_{n+1},x_n\right),b\left(x_{n+1},x_{n-1}\right),\right.\\&\left.b\left(x_n,x_n\right)\right]=g\left[b\left(x_{n-1},x_n\right),b\left(x_{n-1},x_n\right),b\left(x_n,x_{n+1}\right),b\left(x_{n-1},x_{n+1}\right),0\right]\end{aligned}$$

whence comparing with point 1. of Definition 3. we have $p=b\left(x_{n-1},x_n\right)$, $2p+r=b\left(x_{n-1},x_n\right)$, $r=b\left(x_n,x_{n+1}\right)$, $p+2q=b\left(x_{n-1},x_{n+1}\right)$ and $q=0$. We also see that $q\leq 2(p+r)$ is fulfilled because it is obvious. Then according to point 1. of Definition 3. it follows that $b(x_n,x_{n+1}\})\leq\phi\cdot b(x_{n-1},x_n)$, where ϕ belongs to $[0,1)$. Since x_n is different from x_{n-1} for each n we have that ϕ belongs to $(0,1)$. According to Lemma 2.2. it follows that the sequence $\{x_n\}$ is Cauchy in the b-metric space $(X,b,\frac{3}{2})$. Since the introduced b-metric space is complete, we have that the sequence x_n converges to some point u from X. We will prove in the **last step** that u is a fixed point of Q. Putting $x=x_n$, $y=u$ in the given contractive condition we get

$$\begin{aligned}b(x_{n+1},Qu)&\leq g\left[b\left(x_n,u\right),b\left(x_{n+1},x_n\right),b\left(Qu,u\right),b\left(Qu,x_n\right),b\left(u,x_{n+1}\right)\right]\\&\leq g\left[b\left(x_n,u\right),b\left(x_{n+1},x_n\right),b\left(Qu,u\right),\frac{3}{2}b\left(Qu,u\right)+\frac{3}{2}b\left(u,x_n\right),b\left(u,x_{n+1}\right)\right].\end{aligned}$$

By moving to the limit value when n tends to $+\infty$ and using the continuity of the b-metric b and the function g, we get

$$\begin{aligned}b\left(u,Qu\right)\leq&g\left[0,0,b\left(Qu,u\right),\frac{3}{2}b\left(Qu,u\right)+\frac{3}{2}\cdot 0,0\right]\\=&g\left[0,0,b\left(Qu,u\right),\frac{3}{2}b\left(Qu,u\right),0\right].\end{aligned}$$

Since the conditions of point 1. of Definition 3. are obviously met, we get that $b(Qu,u)\leq\phi\cdot 0=0$, for some $\phi\in[0,1)$. From which it follows that $Qu=u$, i.e., that u is a unique fixed point of mapping Q. It also means that u is a unique fixed point of mapping T^m, i.e., that u is a unique fixed point of the mapping T. The proof of Theorem is complete. □

Remark 2.4. On the open problem in [6].

Comparing Definition 4. of the introduction of a generalized Sehgal-Guseman-like contraction in S-metric spaces [6], with the corresponding definitions of Sehgal and Guseman's ordinary contraction given in the works of [25] and [12], we see that it is not in the true spirit of Sehgal-Guseman. This is because the degree-exponent of the mapping T does not depend on the argument x, i.e., it is not of the form that for each x in X there is a positive integer $n(x)$ such that for each $y \in X$ some inequality (relation) is fulfilled. That is why an open problem was set at the end of [6] work. Using the relation of the S-metric to the b-metric, that open problem might have the following simpler (clearer) formulation: Examine the specific conditions of the function g from $\mathcal{D}$ and the mapping of T from the b-metric space (X, b) to itself such that it has a unique fixed point if the condition is fulfilled: For each $x \in X$ there exists a positive integer $n(x)$ such that for each $y \in X$ the following is fulfilled

$$b\left(T^{n(x)}\left(x\right), T^{n(x)}\left(y\right)\right)$$

$$\leq g\left[b\left(x, y\right), b\left(T^{n(x)}x, x\right), b\left(T^{n(x)}x, y\right), b\left(T^{n(x)}y, x\right), b\left(T^{n(x)}y, y\right)\right]$$

Using the works [18] and [19] we have the answer for some functions g from $\mathcal{D}$. While in the general case the problem remains open in both framework S-metric and b-metric spaces. So, if we have that for every x in X there exists a positive integer $n(x)$ such that the above condition holds, then we could say that we have a true generalized Sehgal-Guseman-like contraction.

Finally, we conclude the paper with a list of all 120 possible generalized Sehgal-Guseman-like contractions in the sense of the definition 4. given in the paper by [6]. Due to the connection of the S-metric S with the b-metric b and for technical reasons of simpler notation, the b-metric b participates in our list. So, we take the b-metric space $(X, b, \frac{3}{2})$ created from the given S-metric space (X, S).

$$\{b(x,y), b(Qx,x), b(Qx,y), b(Qy,x), b(Qy,y)\}, \{b(x,y), b(Qx,x), b(Qx,y), b(Qy,y), b(Qy,x)\},$$

$$\{b(x,y), b(Qx,x), b(Qy,x), b(Qx,y), b(Qy,y)\}, \{b(x,y), b(Qx,x), b(Qy,x), b(Qy,y), b(Qx,y)\},$$

$$\{b(x,y), b(Qx,x), b(Qy,y), b(Qx,y), b(Qy,x)\}, \{b(x,y), b(Qx,x), b(Qy,y), b(Qy,x), b(Qx,y)\},$$

$$\{b(x,y), b(Qx,y), b(Qx,x), b(Qy,x), b(Qy,y)\}, \{b(x,y), b(Qx,y), b(Qx,x), b(Qy,y), b(Qy,x)\},$$

$$\{b(x,y), b(Qx,y), b(Qy,x), b(Qx,x), b(Qy,y)\}, \{b(x,y), b(Qx,y), b(Qy,x), b(Qy,y), b(Qx,x)\},$$

$$\{b(x,y), b(Qx,y), b(Qy,y), b(Qx,x), b(Qy,x)\}, \{b(x,y), b(Qx,y), b(Qy,y), b(Qy,x), b(Qx,x)\},$$

$$\{b(x,y), b(Qy,x), b(Qx,x), b(Qx,y), b(Qy,y)\}, \{b(x,y), b(Qy,x), b(Qx,x), b(Qy,y), b(Qx,y)\},$$

$$\{b(x,y), b(Qy,x), b(Qx,y), b(Qx,x), b(Qy,y)\}, \{b(x,y), b(Qy,x), b(Qx,y), b(Qy,y), b(Qx,x)\},$$

$$\{b(x,y), b(Qy,x), b(Qy,y), b(Qx,x), b(Qx,y)\}, \{b(x,y), b(Qy,x), b(Qy,y), b(Qx,y), b(Qx,x)\},$$

$$\{b(x,y), b(Qy,y), b(Qx,x), b(Qx,y), b(Qy,x)\}, \{b(x,y), b(Qy,y), b(Qx,x), b(Qy,x), b(Qx,y)\},$$

$$\{b(x,y), b(Qy,y), b(Qx,y), b(Qx,x), b(Qy,x)\}, \{b(x,y), b(Qy,y), b(Qx,y), b(Qy,x), b(Qx,x)\},$$

$$\{b(x,y), b(Qy,y), b(Qy,x), b(Qx,x), b(Qx,y)\}, \{b(x,y), b(Qy,y), b(Qy,x), b(Qx,y), b(Qx,x)\},$$

$$\{b(Qx,x), b(x,y), b(Qx,y), b(Qy,x), b(Qy,y)\}, \{b(Qx,x), b(x,y), b(Qx,y), b(Qy,y), b(Qy,x)\},$$

$$\{b(Qx,x), b(x,y), b(Qy,x), b(Qx,y), b(Qy,y)\}, \{b(Qx,x), b(x,y), b(Qy,x), b(Qy,y), b(Qx,y)\},$$

$$\{b(Qx,x),b(x,y),b(Qy,y),b(Qx,y),b(Qy,x)\},\{b(Qx,x),b(x,y),b(Qy,y),b(Qy,x),b(Qx,y)\},$$
$$\{b(Qx,x),b(Qx,y),b(x,y),b(Qy,x),b(Qy,y)\},\{b(Qx,x),b(Qx,y),b(x,y),b(Qy,y),b(Qy,x)\},$$
$$\{b(Qx,x),b(Qx,y),b(Qy,x),b(x,y),b(Qy,y)\},\{b(Qx,x),b(Qx,y),b(Qy,x),b(Qy,y),b(x,y)\},$$
$$\{b(Qx,x),b(Qx,y),b(Qy,y),b(x,y),b(Qy,x)\},\{b(Qx,x),b(Qx,y),b(Qy,y),b(Qy,x),b(x,y)\},$$
$$\{b(Qx,x),b(Qy,x),b(x,y),b(Qx,y),b(Qy,y)\},\{b(Qx,x),b(Qy,x),b(x,y),b(Qy,y),b(Qx,y)\},$$
$$\{b(Qx,x),b(Qy,x),b(Qx,y),b(x,y),b(Qy,y)\},\{b(Qx,x),b(Qy,x),b(Qx,y),b(Qy,y),b(x,y)\},$$
$$\{b(Qx,x),b(Qy,x),b(Qy,y),b(x,y),b(Qx,y)\},\{b(Qx,x),b(Qy,x),b(Qy,y),b(Qx,y),b(x,y)\},$$
$$\{b(Qx,x),b(Qy,y),b(x,y),b(Qx,y),b(Qy,x)\},\{b(Qx,x),b(Qy,y),b(x,y),b(Qy,x),b(Qx,y)\},$$
$$\{b(Qx,x),b(Qy,y),b(Qx,y),b(x,y),b(Qy,x)\},\{b(Qx,x),b(Qy,y),b(Qx,y),b(Qy,x),b(x,y)\},$$
$$\{b(Qx,x),b(Qy,y),b(Qy,x),b(x,y),b(Qx,y)\},\{b(Qx,x),b(Qy,y),b(Qy,x),b(Qx,y),b(x,y)\},$$
$$\{b(Qx,y),b(x,y),b(Qx,x),b(Qy,x),b(Qy,y)\},\{b(Qx,y),b(x,y),b(Qx,x),b(Qy,y),b(Qy,x)\},$$
$$\{b(Qx,y),b(x,y),b(Qy,x),b(Qx,x),b(Qy,y)\},\{b(Qx,y),b(x,y),b(Qy,x),b(Qy,y),b(Qx,x)\},$$
$$\{b(Qx,y),b(x,y),b(Qy,y),b(Qx,x),b(Qy,x)\},\{b(Qx,y),b(x,y),b(Qy,y),b(Qy,x),b(Qx,x)\},$$
$$\{b(Qx,y),b(Qx,x),b(x,y),b(Qy,x),b(Qy,y)\},\{b(Qx,y),b(Qx,x),b(x,y),b(Qy,y),b(Qy,x)\},$$
$$\{b(Qx,y),b(Qx,x),b(Qy,x),b(x,y),b(Qy,y)\},\{b(Qx,y),b(Qx,x),b(Qy,x),b(Qy,y),b(x,y)\},$$
$$\{b(Qx,y),b(Qx,x),b(Qy,y),b(x,y),b(Qy,x)\},\{b(Qx,y),b(Qx,x),b(Qy,y),b(Qy,x),b(x,y)\},$$
$$\{b(Qx,y),b(Qy,x),b(x,y),b(Qx,x),b(Qy,y)\},\{b(Qx,y),b(Qy,x),b(x,y),b(Qy,y),b(Qx,x)\},$$
$$\{b(Qx,y),b(Qy,x),b(Qx,x),b(x,y),b(Qy,y)\},\{b(Qx,y),b(Qy,x),b(Qx,x),b(Qy,y),b(x,y)\},$$
$$\{b(Qx,y),b(Qy,x),b(Qy,y),b(x,y),b(Qx,x)\},\{b(Qx,y),b(Qy,x),b(Qy,y),b(Qx,x),b(x,y)\},$$
$$\{b(Qx,y),b(Qy,y),b(x,y),b(Qx,x),b(Qy,x)\},\{b(Qx,y),b(Qy,y),b(x,y),b(Qy,x),b(Qx,x)\},$$
$$\{b(Qx,y),b(Qy,y),b(Qx,x),b(x,y),b(Qy,x)\},\{b(Qx,y),b(Qy,y),b(Qx,x),b(Qy,x),b(x,y)\},$$
$$\{b(Qx,y),b(Qy,y),b(Qy,x),b(x,y),b(Qx,x)\},\{b(Qx,y),b(Qy,y),b(Qy,x),b(Qx,x),b(x,y)\},$$
$$\{b(Qy,x),b(x,y),b(Qx,x),b(Qx,y),b(Qy,y)\},\{b(Qy,x),b(x,y),b(Qx,x),b(Qy,y),b(Qx,y)\},$$
$$\{b(Qy,x),b(x,y),b(Qx,y),b(Qx,x),b(Qy,y)\},\{b(Qy,x),b(x,y),b(Qx,y),b(Qy,y),b(Qx,x)\},$$
$$\{b(Qy,x),b(x,y),b(Qy,y),b(Qx,x),b(Qx,y)\},\{b(Qy,x),b(x,y),b(Qy,y),b(Qx,y),b(Qx,x)\},$$
$$\{b(Qy,x),b(Qx,x),b(x,y),b(Qx,y),b(Qy,y)\},\{b(Qy,x),b(Qx,x),b(x,y),b(Qy,y),b(Qx,y)\},$$
$$\{b(Qy,x),b(Qx,x),b(Qx,y),b(x,y),b(Qy,y)\},\{b(Qy,x),b(Qx,x),b(Qx,y),b(Qy,y),b(x,y)\},$$
$$\{b(Qy,x),b(Qx,x),b(Qy,y),b(x,y),b(Qx,y)\},\{b(Qy,x),b(Qx,x),b(Qy,y),b(Qx,y),b(x,y)\},$$
$$\{b(Qy,x),b(Qx,y),b(x,y),b(Qx,x),b(Qy,y)\},\{b(Qy,x),b(Qx,y),b(x,y),b(Qy,y),b(Qx,x)\},$$
$$\{b(Qy,x),b(Qx,y),b(Qx,x),b(x,y),b(Qy,y)\},\{b(Qy,x),b(Qx,y),b(Qx,x),b(Qy,y),b(x,y)\},$$
$$\{b(Qy,x),b(Qx,y),b(Qy,y),b(x,y),b(Qx,x)\},\{b(Qy,x),b(Qx,y),b(Qy,y),b(Qx,x),b(x,y)\},$$
$$\{b(Qy,x),b(Qy,y),b(x,y),b(Qx,x),b(Qx,y)\},\{b(Qy,x),b(Qy,y),b(x,y),b(Qx,y),b(Qx,x)\},$$
$$\{b(Qy,x),b(Qy,y),b(Qx,x),b(x,y),b(Qx,y)\},\{b(Qy,x),b(Qy,y),b(Qx,x),b(Qx,y),b(x,y)\},$$
$$\{b(Qy,x),b(Qy,y),b(Qx,y),b(x,y),b(Qx,x)\},\{b(Qy,x),b(Qy,y),b(Qx,y),b(Qx,x),b(x,y)\},$$
$$\{b(Qy,y),b(x,y),b(Qx,x),b(Qx,y),b(Qy,x)\},\{b(Qy,y),b(x,y),b(Qx,x),b(Qy,x),b(Qx,y)\},$$
$$\{b(Qy,y),b(x,y),b(Qx,y),b(Qx,x),b(Qy,x)\},\{b(Qy,y),b(x,y),b(Qx,y),b(Qy,x),b(Qx,x)\},$$
$$\{b(Qy,y),b(x,y),b(Qy,x),b(Qx,x),b(Qx,y)\},\{b(Qy,y),b(x,y),b(Qy,x),b(Qx,y),b(Qx,x)\},$$
$$\{b(Qy,y),b(Qx,x),b(x,y),b(Qx,y),b(Qy,x)\},\{b(Qy,y),b(Qx,x),b(x,y),b(Qy,x),b(Qx,y)\},$$
$$\{b(Qy,y),b(Qx,x),b(Qx,y),b(x,y),b(Qy,x)\},\{b(Qy,y),b(Qx,x),b(Qx,y),b(Qy,x),b(x,y)\},$$

$$\{b(Qy,y),b(Qx,x),b(Qy,x),b(x,y),b(Qx,y)\},\{b(Qy,y),b(Qx,x),b(Qy,x),b(Qx,y),b(x,y)\},$$
$$\{b(Qy,y),b(Qx,y),b(x,y),b(Qx,x),b(Qy,x)\},\{b(Qy,y),b(Qx,y),b(x,y),b(Qy,x),b(Qx,x)\},$$
$$\{b(Qy,y),b(Qx,y),b(Qx,x),b(x,y),b(Qy,x)\},\{b(Qy,y),b(Qx,y),b(Qx,x),b(Qy,x),b(x,y)\},$$
$$\{b(Qy,y),b(Qx,y),b(Qy,x),b(x,y),b(Qx,x)\},\{b(Qy,y),b(Qx,y),b(Qy,x),b(Qx,x),b(x,y)\},$$
$$\{b(Qy,y),b(Qy,x),b(x,y),b(Qx,x),b(Qx,y)\},\{b(Qy,y),b(Qy,x),b(x,y),b(Qx,y),b(Qx,x)\},$$
$$\{b(Qy,y),b(Qy,x),b(Qx,x),b(x,y),b(Qx,y)\},\{b(Qy,y),b(Qy,x),b(Qx,x),b(Qx,y),b(x,y)\},$$
$$\{b(Qy,y),b(Qy,x),b(Qx,y),b(x,y),b(Qx,x)\},\{b(Qy,y),b(Qy,x),b(Qx,y),b(Qx,x),b(x,y)\}.$$

The previous list is easily translated into a new list by equating each specified permutation from the given list to $[p,p+r,r,2p+q,q]$ and eliminating the three variables p,q,r gives the relation between the components of each of the 120 permutation. Let us show this with an example,

$$[b\left(x,y\right),b\left(Qx,x\right),b\left(Qx,y\right),b\left(Qy,x\right),b\left(Qy,y\right)].$$

Equating now $[b\left(x,y\right),b\left(Qx,x\right),b\left(Qx,y\right),b\left(Qy,x\right),b\left(Qy,y\right)]$ with $[p,2p+r,r,p+2q,q]$ we have the following five equalities:

$b\left(x,y\right)=p,$
$b\left(Qx,x\right)=2p+r,$
$b\left(Qx,y\right)=r,$
$b\left(Qy,x\right)=p+2q,$
$b\left(Qy,y\right)=q.$

Whence follows from the first three equations that $b(Qx,x)=2b\left(x,y\right)+b\left(Qx,y\right)$, i.e., $2b\left(x,y\right)=b(Qx,x)-b\left(Qx,y\right)$. While from the last two equalities we get $b\left(Qy,x\right)=b\left(x,y\right)+2b\left(Qy,y\right)$, i.e., $b\left(x,y\right)=b\left(Qy,x\right)-2b\left(Qy,y\right)$. Hence, we have obtained second and first line in the new list. That new list created from the first list looks like this:

$$b(x,y)=b(Qy,x)-2b(Qy,y),\ b(x,y)=b(Qy,y)-2b(Qy,x),$$
$$2b(x,y)=b(Qx,x)-b(Qy,x),\ 2b(x,y)=b(Qx,x)-b(Qy,x),$$
$$2b(x,y)=b(Qx,x)-b(Qy,y),\ 2b(x,y)=b(Qx,x)-b(Qy,y),$$
$$b(x,y)=b(Qy,x)-2b(Qy,y),\ b(x,y)=b(Qy,y)-2b(Qy,x),$$
$$b(x,y)=b(Qx,x)-2b(Qy,y),\ b(x,y)=b(Qy,y)-2b(Qx,x),$$
$$b(x,y)=b(Qx,x)-2b(Qy,x),\ b(x,y)=b(Qy,x)-2b(Qx,x),$$
$$2b(x,y)=b(Qy,x)-b(Qx,x),\ 2b(x,y)=b(Qy,x)-b(Qx,x),$$
$$b(x,y)=b(Qx,x)-2b(Qy,y),\ b(x,y)=b(Qy,y)-2b(Qx,x),$$
$$2b(x,y)=b(Qy,x)-b(Qy,y),\ 2b(x,y)=b(Qy,x)-b(Qy,y),$$
$$2b(x,y)=b(Qy,y)-b(Qx,x),\ 2b(x,y)=b(Qy,y)-b(Qx,x),$$
$$b(x,y)=b(Qx,x)-2b(Qy,x),\ b(x,y)=b(Qy,x)-2b(Qx,x),$$
$$2b(x,y)=b(Qy,y)-b(Qy,x),\ 2b(x,y)=b(Qy,y)-b(Qy,x),$$
$$b(Qy,x)-2b(Qy,y)=b(Qx,x),\ b(Qy,y)-2b(Qy,x)=b(Qx,x),$$
$$b(x,y)-b(Qy,x)=2b(Qx,x),\ b(x,y)-b(Qy,x)=2b(Qx,x),$$
$$b(x,y)-b(Qy,y)=2b(Qx,x),\ b(x,y)-b(Qy,y)=2b(Qx,x),$$

$$b(Qy,x)-2b(Qy,y)=b(Qx,x),\ b(Qy,y)-2b(Qy,x)=b(Qx,x),$$

$$b(x,y)-2b(Qy,y)=b(Qx,x),\ b(Qy,y)-2b(x,y)=b(Qx,x),$$

$$b(x,y)-2b(Qy,x)=b(Qx,x),\ b(Qy,x)-2b(x,y)=b(Qx,x),$$

$$b(Qy,x)-b(x,y)=2b(Qx,x),\ b(Qy,x)-b(x,y)=2b(Qx,x),$$

$$b(x,y)-2b(Qy,y)=b(Qx,x),\ b(Qy,y)-2b(x,y)=b(Qx,x),$$

$$b(Qy,x)-b(Qy,y)=2b(Qx,x),\ b(Qy,x)-b(Qy,y)=2b(Qx,x),$$

$$b(Qy,y)-b(x,y)=2b(Qx,x),\ b(Qy,y)-b(x,y)=2b(Qx,x),$$

$$b(x,y)-2b(Qy,x)=b(Qx,x),\ b(Qy,x)-2b(x,y)=b(Qx,x),$$

$$b(Qy,y)-b(Qy,x)=2b(Qx,x),\ b(Qy,y)-b(Qy,x)=2b(Qx,x),$$

$$b(Qx,x)+2b(Qy,x)-b(x,y)=4b(Qy,y),\ b(Qx,x)+2b(Qy,y)-b(x,y)=4b(Qy,x),$$

$$2b(Qx,x)+b(Qy,x)-b(x,y)=4b(Qy,y),\ b(Qy,x)+2b(Qy,y)-b(x,y)=4b(Qx,x),$$

$$2b(Qx,x)+b(Qy,y)-b(x,y)=4b(Qy,x),\ 2b(Qy,x)+b(Qy,y)-b(x,y)=4b(Qx,x),$$

$$-b(Qx,x)+2b(Qy,x)+b(x,y)=4b(Qy,y),\ \ -b(Qx,x)+2b(Qy,y)+b(x,y)=4b(Qy,x),$$

$$-b(Qx,x)+b(Qy,x)+2b(x,y)=4b(Qy,y),\ 4b(x,y)=-b(Qx,x)+b(Qy,x)+2b(Qy,y),$$

$$-b(Qx,x)+b(Qy,y)+2b(x,y)=4b(Qy,x),\ 4b(x,y)=-b(Qx,x)+2b(Qy,x)+b(Qy,y),$$

$$2b(Qx,x)-b(Qy,x)+b(x,y)=4b(Qy,y),\ \ -b(Qy,x)+2b(Qy,y)+b(x,y)=4b(Qx,x),$$

$$b(Qx,x)-b(Qy,x)+2b(x,y)=4b(Qy,y),\ 4b(x,y)=b(Qx,x)-b(Qy,x)+2b(Qy,y),$$

$$-b(Qy,x)+b(Qy,y)+2b(x,y)=4b(Qx,x),\ 4b(x,y)=2b(Qx,x)-b(Qy,x)+b(Qy,y),$$

$$2b(Qx,x)-b(Qy,y)+b(x,y)=4b(Qy,x),\ 2b(Qy,x)-b(Qy,y)+b(x,y)=4b(Qx,x),$$

$$b(Qx,x)-b(Qy,y)+2b(x,y)=4b(Qy,x),\ 4b(x,y)=b(Qx,x)+2b(Qy,x)-b(Qy,y),$$

$$b(Qy,x)-b(Qy,y)+2b(x,y)=4b(Qx,x),\ 4b(x,y)=2b(Qx,x)+b(Qy,x)-b(Qy,y),$$

$$b(x,y)-b(Qx,x)=2b(Qy,x),\ b(x,y)-b(Qx,x)=2b(Qy,x),$$

$$b(Qx,x)-2b(Qy,y)=b(Qy,x),\ b(Qy,y)-2b(Qx,x)=b(Qy,x),$$

$$b(x,y)-b(Qy,y)=2b(Qy,x),\ b(x,y)-b(Qy,y)=2b(Qy,x),$$

$$b(Qx,x)-b(x,y)=2b(Qy,x),\ b(Qx,x)-b(x,y)=2b(Qy,x),$$

$$b(x,y)-2b(Qy,y)=b(Qy,x),\ b(Qy,y)-2b(x,y)=b(Qy,x),$$

$$b(Qx,x)-b(Qy,y)=2b(Qy,x),\ b(Qx,x)-b(Qy,y)=2b(Qy,x),$$

$$b(Qx,x)-2b(Qy,y)=b(Qy,x),\ b(Qy,y)-2b(Qx,x)=b(Qy,x),$$

$$b(x,y)-2b(Qy,y)=b(Qy,x),\ b(Qy,y)-2b(x,y)=b(Qy,x),$$

$$b(x,y)-2b(Qx,x)=b(Qy,x),\ b(Qx,x)-2b(x,y)=b(Qy,x),$$

$$b(Qy,y)-b(x,y)=2b(Qy,x),\ b(Qy,y)-b(x,y)=2b(Qy,x),$$

$$b(Qy,y)-b(Qx,x)=2b(Qy,x),\ b(Qy,y)-b(Qx,x)=2b(Qy,x),$$

$$b(x,y)-2b(Qx,x)=b(Qy,x),\ b(Qx,x)-2b(x,y)=b(Qy,x),$$

$$b(x,y)-b(Qx,x)=2b(Qy,y),\ b(x,y)-b(Qx,x)=2b(Qy,y),$$

$$b(Qy,y)=b(Qx,x)-2b(Qy,x),\ b(Qy,y)=b(Qy,x)-2b(Qx,x),$$

$$b(x,y)-b(Qy,x)=2b(Qy,y),\ b(x,y)-b(Qy,x)=2b(Qy,y),$$

$$b(Qx,x)-b(x,y)=2b(Qy,y),\ b(Qx,x)-b(x,y)=2b(Qy,y),$$

$$b(x,y)-2b(Qy,x)=b(Qy,y),\ b(Qy,x)-2b(x,y)=b(Qy,y),$$

$$2b(Qy,y)=b(Qx,x)-b(Qy,x),\ 2b(Qy,y)=b(Qx,x)-b(Qy,x),$$

$$b(Qy,y)=b(Qx,x)-2b(Qy,x),\ b(Qy,y)=b(Qy,x)-2b(Qx,x),$$
$$b(x,y)-2b(Qy,x)=b(Qy,y),\ b(Qy,x)-2b(x,y)=b(Qy,y),$$
$$b(x,y)-2b(Qx,x)=b(Qy,y),\ b(Qx,x)-2b(x,y)=b(Qy,y),$$
$$b(Qy,x)-b(x,y)=2b(Qy,y),\ b(Qy,x)-b(x,y)=2b(Qy,y),$$
$$2b(Qy,y)=b(Qy,x)-b(Qx,x),\ 2b(Qy,y)=b(Qy,x)-b(Qx,x),$$
$$b(x,y)-2b(Qx,x)=b(Qy,y),\ b(Qx,x)-2b(x,y)=b(Qy,y).$$

We read each of the previous 120 permutations, i.e., general Sehgal-Guseman-similar-contractions in accordance with the markings in the previous part of the paper.

2.2. Consequences and Outlook. Since we have indicated that by replacing the S-metric with an adequate b-metric, the main results from [6] work can be significantly improved and simplified in some aspects, it is natural to find precedents that support this. It is not difficult to verify that all the examples, consequences and applications given in the paper [6] achieve the desired goal by simply modifying the transition from the S-metric to the b-metric.

Therefore, instead of the S-metric space (X,S), the b-metric space $(X,b,\frac{3}{2})$ enters the scene, where the b-metric b is a continuous function with two variables. Since b-metric spaces are much more well-known and elaborated in the literature than S-metric spaces, we gave them priority in achieving the results in this work. For more details on b-metric spaces see works such as: [2, 9, 10, 17]-[19, 23, 30]. Let us note that by some properties b-metric spaces are close to ordinary metric properties (see for example Theorem 4 in applications).

Finally, we conclude the paper with the following open question: Is it possible to prove or disprove the fact of the existence of a unique fixed point in some way, perhaps by using artificial intelligence, a computer method or a special procedure for each permutation from the first list.

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Received : *May* 2024
Accepted : *June* 2024