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Remarks on "On new orthogonal contractions in b-metric spaces" and "On orthogonal partial b-metric spaces with an application"

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Abstract. The results published by O. Yamaod and W. Sintunavarat in [On new orthogonal contractions in b-metric spaces, Intern. J. Pure Math., 5, **2018**, 37–40], and by K. Javed, H. Aydi, F. Uddin and M. Arshad in [On orthogonal partial b-metric spaces with an application, J. Mathematics (Hindawi), **2021**, Article ID 6692063, 7 pages] are discussed. First of all, some formulations that are not precise in these papers are commented. Then these results are extended and unified from the case of orthogonal b-metric and orthogonal partial b-metric spaces to the wider framework of orthogonal b-metric-like spaces. Moreover, some results are generalized by proving that the contraction parameter may belong to the wider set $[0, 1)$.

1. Introduction and preliminaries

1.1. *b*-metric-like spaces. In [1], the authors introduced the following concept, thus generalizing the notions of b-metric spaces [3, 5], partial b-metric spaces [12] and metric-like spaces [2].

Definition 1.1. [1] A b-metric-like on a nonempty set X is a function $d : X \times X \to [0, \infty)$ if there exists a constant $s \geq 1$ such that for all $x, y, z \in X$, the following conditions hold:

(1) $d(x, y) = 0$ implies $x = y$;
(2) $d(x, y) = d(y, x)$;

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(3) $d(x,y) \le s(d(x,z) + d(z,y))$.

Then (X,d,s) is called a b-metric-like space.

We recall in the following diagram the relationship between the mentioned classes of spaces (where arrows stand for implications).

$$\begin{array}{ccccc} \text{Metric space} & \to & \text{Partial metric space} & \to & \text{Metric-like space} \\ \downarrow & & \downarrow & & \downarrow \\ b\text{-metric space} & \to & \text{Partial } b\text{-metric space} & \to & b\text{-metric-like space} \end{array} \tag{1}$$

This diagram remains valid if the name of each type of spaces is preceded by "orthogonal" (see definition in Subsection 1.2), "partially ordered" or alike. In this paper we will present results in the most general class of (orthogonal) b-metric-like spaces.

The notions of convergent and Cauchy sequences were introduced in b-metric-like spaces as follows.

Definition 1.2. [1] Let (X,d,s) be a b-metric-like space and $\{x_n\}$ be a sequence in X.

(1) The sequence $\{x_n\}$ is said to converge to $x \in X$ if $\lim_{n\to\infty} d(x,x_n) = d(x,x)$.
(2) $\{x_n\}$ is a Cauchy sequence if $\lim_{m,n\to\infty} d(x_m,x_n)$ exists and is finite.
(3) The space (X,d,s) is said to be complete if every Cauchy sequence $\{x_n\}$ in X converges to some $x \in X$ such that $\lim_{m,n\to\infty} d(x_m,x_n) = d(x,x) = \lim_{n\to\infty} d(x_n,x)$.

Several results on (common) fixed points in the framework of b-metric-like spaces were obtained by various authors in subsequent years. In proofs of some of these results the following lemma is often useful.

Lemma 1.1. *Let (X,d,s) be a b-metric-like space and $T : X \to X$ be a mapping. Suppose that $\{x_n\}$ is a sequence in X defined by $x_0 \in X$ and $x_n = Tx_{n-1}$, $n \in \mathbb{N}$, satisfying*

$$d(x_n,x_{n+1}) \le \lambda\, d(x_{n-1},x_n), \tag{2}$$

for some constant $\lambda \in [0,1/s)$ and for all $n \in \mathbb{N}$. Then $\{x_n\}$ is a Cauchy sequence in (X,d,s).

Proof. Using (2) we have

$$d(x_n,x_{n+1}) \le \lambda d(x_{n-1},x_n) \le \lambda^2 d(x_{n-2},x_{n-1}) \le \cdots \le \lambda^n d(x_0,x_1),$$

implying that, for $m, n \in \mathbb{N}$ with $m < n$,

$$\begin{aligned} d(x_m, x_n) &\le s[d(x_m, x_{m+1}) + d(x_{m+1}, x_n)] \\ &\le sd(x_m, x_{m+1}) + s^2[d(x_{m+1}, x_{m+2}) + d(x_{m+2}, x_n)] \\ &\le \cdots \\ &\le sd(x_m, x_{m+1}) + s^2 d(x_{m+1}, x_{m+2}) + \cdots \\ &\quad + s^{n-m-1} d(x_{n-2}, x_{n-1}) + s^{n-m-1} d(x_{n-1}, x_n) \\ &\le s\lambda^m d(x_0, x_1) + s^2\lambda^{m+1} d(x_0, x_1) + \cdots \\ &\quad + s^{n-m-1}\lambda^{n-2} d(x_0, x_1) + s^{n-m-1}\lambda^{n-1} d(x_0, x_1) \\ &\le s\lambda^m[1 + s\lambda + \cdots + s^{n-m-2}\lambda^{n-m-2} + s^{n-m-1}\lambda^{n-m-1}] d(x_0, x_1) \\ &\le s\lambda^m\Big[\sum_{i=0}^{\infty}(s\lambda)^i\Big] d(x_0, x_1) = \frac{s\lambda^m}{1 - s\lambda} d(x_0, x_1) \to 0 \quad (m \to \infty). \end{aligned}$$

This means that $\{x_n\} = \{T^n x_0\}$ is a Cauchy sequence in (X, d, s). □

Remark 1.2. We note that in several papers (including [14, Theorem 3.5]) proofs of similar propositions are often not fully correct. Namely, they prove that $\frac{s^p\lambda^m}{s-\lambda}$ tend to zero as $m \to \infty$ *for each fixed* $p \in \mathbb{N}$, where λ is a given number from $(0, 1)$ and s, for example, is strictly greater than 1 – in fact, one should need that this convergence is *uniform* in $p \in \mathbb{N}$ (or, as was done in our case, that $d(x_m, x_n)$ is considered and that the limit when *both* parameters m, n tend to ∞ is proved to be equal to 0).

1.2. Orthogonal spaces. Now we state the basic properties of the so-called orthogonal relation introduced on a non-empty set X. It was first introduced by M. E. Gordji, M. Rameani, M. De La Sen and Y. J. Cho in [10]. A binary relation, denoted as $\perp$, on a non-empty set X is said to be orthogonal if there exists an element $a \in X$ such that it is in the relation $\perp$ with every $y \in X$ or that every $y \in X$ is in the relation with it. If X is also provided with some metric d then $(X, \perp, d)$ is said to be an orthogonal metric space. Orthogonal partial metric spaces, orthogonal b-metric spaces, and similar generalized orthogonal spaces are defined in the same manner. For more examples of these notions see, for example, [7, 8, 9, 10, 11, 14].

In all cases we say that $(X, \perp)$ is an orthogonal or O-set. A sequence $\{x_n\}$ in a given O-set is said to be an O-sequence if we have that $x_n \perp x_{n+1}$ for each $n \in N$ or $x_{n+1} \perp x_n$ for each $n \in N$. In the definitions of convergent and Cauchy sequences it is supposed that sequences considered in Definition 1.2 are O-sequences. Orthogonal (generalized) metric space is said to be orthogonally complete (O-complete) if every Cauchy O-sequence in it is convergent.

A self-mapping T of a (generalized) orthogonal metric space $(X, \perp, d)$ is said to be orthogonally continuous (or $\perp$-continuous, OC) at $x \in X$ if for each O-sequence

$\{x_n\}$ in X with $x_n \to x$ we have $Tx_n \to Tx$. Also, T is said to be $\perp$-continuous on X if T is $\perp$-continuous at each $x \in X$. The mapping T is said to be $\perp$-preserving (OP) if $Tx \perp Ty$ whenever $x \perp y$. Further, T is said to be weakly $\perp$-preserving if $Tx \perp Ty$ or $Ty \perp Tx$ whenever $x \perp y$.

The main goal of this paper is to review the results of two recent papers. Namely, we discuss the results published by O. Yamaod and W. Sintunavarat in [14], and by K. Javed, H. Aydi, F. Uddin and M. Arshad in [11]. First of all, we comment on some formulations in these papers that are not precise. Then we unify and extend these results from the case of orthogonal b-metric and orthogonal partial b-metric spaces to the wider framework of orthogonal b-metric-like spaces. Moreover, some of the results are generalized by proving that the contraction parameter may belong to the wider set $[0, 1)$. Examples are presented in order to illustrate the obtained results.

2. Results

2.1. Comments on the results from [11] and [14]. In Definition 3.1 of the paper [14] the authors introduce so-called s-orthogonal contraction claiming that it generalizes the ordinary orthogonal contraction introduced in the paper [10]. However, this is not the case because from the condition $s \cdot d(fx, fy) \leq \lambda \cdot d(x, y)$ of the definition of s-orthogonal contraction it follows, due to $s \geq 1$, that $d(fx, fy) \leq \lambda \cdot d(x, y)$ which represents the definition of ordinary orthogonal contraction. Also, in Definitions 3.2 and 3.3 of the same paper, the index s is redundant because obviously the terms $\perp_s$-continuous and O_s-complete are the same as the corresponding terms introduced in [10] for orthogonal metric spaces. As for the s-orthogonal contraction defined in the paper [14], we have that it is also an ordinary orthogonal contraction. This follows from the fact that $d(fx, fy) \leq s \cdot d(fx, fy)$ since $s \geq 1$. Ordinary orthogonal contraction is different from s-contraction in the case $s > 1$ as can be shown by simple examples.

We also mention in this remark that some things in the paper [11] were not given correctly. We note this primarily because of the growing number of young researchers working in this field of generalized metric spaces. Such are, for example, all four Definitions 10–13. The readers can see accurate variants of the above definitions, e.g., in [12] or [13]. In Examples 2 and 3 of [11] the given sets are not O-sets since there is no element in them which is orthogonal to all other elements of the spaces (or vice versa). Moreover, in Example 3 it should be $s = 2$ (not $s = 4$ as it is written). On the other hand, in the formulation of Theorem 2, it is not written that the condition (23) has to hold only when $\kappa_1 \perp \kappa_2$.

2.2. Generalizations. Now we will combine the results from [11] and [14] for orthogonal partial b-metric spaces and orthogonal b-metric spaces and prove them in the framework of orthogonal b-metric like spaces. Thus, we improve and generalize the results from these two papers to the broadest class of generalized orthogonal metric spaces. Besides that, in the special case on Banach-type contractive condition, the new result will be proved for a wider range of the contraction parameter, namely λ will belong to the whole $[0,1)$.

First of all, let us note that apparently Theorem 1 from [**11**] is a direct consequence of Theorem 2 of this article. A careful consideration shows that Theorem 1 from [**14**], but formulated in the framework of orthogonal partial b-metric spaces, is also a consequence of Theorem 2 from [**11**]. Therefore, our next theorem is a unification of the results from [**11**] and [**14**], but now in the class of orthogonal b-metric-like spaces.

Theorem 2.1. *Let $(X,\perp,d,s)$ be an O-complete orthogonal b-metric-like space and let T be an OP and OC mapping from X to itself such that there exists a $\lambda\in[0,1/s)$ with the property that for all x,y from X with $x\perp y$, the following contractive condition holds*

$$d(Tx,Ty)\le\lambda\max\{d(x,y),d(x,Tx),d(y,Ty)\}. \tag{3}$$

Then T has a unique fixed point $x^*\in X$ and $d(x^*,x^*)=0$ holds.

Proof. Since the orthogonal b-metric-like space differs from orthogonal partial b-metric space only in axiom (1) of Definition 1.1, the proof of our theorem in this case does not differ much from the corresponding proof of [**11**, Theorem 2]. The key is Lemma 1.1. Namely, as in [**11**], it can be obtained that $d(x_{n+1},x_n)\le\lambda\, d(x_n,x_{n-1})$ for each $n\in\mathbb{N}$, and then, according to the stated lemma, it follows that $\lim_{m,n\to\infty}d(x_m,x_n)=0$. Now from the O-completeness of the space $(X,\perp,d,s)$ it follows that there exists a (unique) point $x^*\in X$ such that

$$\lim_{m,n\to\infty}d(x_m,x_n)=\lim_{n\to\infty}d(x_n,x^*)=d(x^*,x^*)=0. \tag{4}$$

From the OC-property of the mapping T, it follows that $Tx^*=x^*$. The proof of the uniqueness of fixed point is the same as in the proof of [**11**, Theorem 2]. The condition $d(x^*,x^*)=0$ was already obtained in (4). □

Corollary 2.2. *Let $(X,\perp,d,s)$ be an O-complete orthogonal b-metric-like space and let T be an OP and OC mapping from X to itself such that there exist nonnegative constants a,b,c with $a+b+c<1/s$ with the property that for all $x,y\in X$ with $x\perp y$, the following Reich-type contractive condition holds*

$$d(Tx,Ty)\le a\,d(x,y)+b\,d(x,Tx)+c\,d(y,Ty). \tag{5}$$

Then T has a unique fixed point $x^*\in X$ and $d(x^*,x^*)=0$ holds.

Corollary 2.3. *The results of Theorem 2.1 (and Corollary 2.2) remain valid in each (orthogonal) space from Table (1).*

Example 2.1. Let $X = [0, \infty)$ and let an orthogonal relation $\perp$ be defined on X by $x \perp y$ if and only if $xy \leq x$, i.e., $x = 0$ or $y \leq 1$ (orthogonal elements are 0 and 1). Let $d : X \times X \to [0, \infty)$ be defined by $d(x, y) = (\max\{x, y\})^2$; then d is a b-metric-like on X with parameter $s = 2$ (see [1]). It is easy to see that $(X, \perp, d, 2)$ is an O-complete orthogonal b-metric-like space.

Consider the mapping $T : X \to X$ given by

$$Tx = \begin{cases} \frac{1}{2}x, & 0 \leq x \leq 3, \\ 0, & x > 3. \end{cases}$$

It is easy to check (see, e.g., [8, Example 2.4]) that T is an OP and OC selfmap of X. In order to check the condition (5), choose $a = b = c = \frac{1}{8}$ (condition $a + b + c < \frac{1}{s}$ is satisfied) and $x, y \in X$ with $x \perp y$, and consider the following possibilities.

1° $x = 0$, $0 \leq y \leq 3$. Then $Tx = 0$, $Ty = \frac{1}{2}y$ and

$$d(Tx, Ty) = \frac{1}{4}y^2 = \frac{1}{8}y^2 + \frac{1}{8} \cdot 0 + \frac{1}{8}y^2 = a\,d(x, y) + b\,d(x, Tx) + c\,d(y, Ty).$$

2° $x = 0$, $y > 3$. Then $Tx = 0$, $Ty = 0$ and relation (5) is trivially satisfied.

3° $y \leq 1$, $0 < x \leq 3$. Then $Tx = \frac{1}{2}x$, $Ty = \frac{1}{2}y$ and $d(Tx, Ty) = \frac{1}{4}(\max\{x, y\})^2$. Consider the following two subcases

3.1° $x \leq y$. Then

$$d(Tx, Ty) = \frac{1}{4}y^2 \leq \frac{1}{8}y^2 + \frac{1}{8}x^2 + \frac{1}{8}y^2 = a\,d(x, y) + b\,d(x, Tx) + c\,d(y, Ty).$$

3.2° $x > y$. Then

$$d(Tx, Ty) = \frac{1}{4}x^2 \leq \frac{1}{8}x^2 + \frac{1}{8}x^2 + \frac{1}{8}y^2 = a\,d(x, y) + b\,d(x, Tx) + c\,d(y, Ty).$$

4° $y \leq 1$, $x > 3$. Then $Tx = 0$, $Ty = \frac{1}{2}y$ and

$$\begin{aligned} d(Tx, Ty) = \frac{1}{4}y^2 &\leq \frac{1}{8}y^2 + \frac{1}{8}y^2 + \frac{1}{8}y^2 \leq \frac{1}{8}x^2 + \frac{1}{8}x^2 + \frac{1}{8}y^2 \\ &= a\,d(x, y) + b\,d(x, Tx) + c\,d(y, Ty). \end{aligned}$$

Thus, in all possible cases, condition (5) is fulfilled and T satisfies all assumptions of Corollary 2.2 and, hence, it has a unique fixed point x^* satisfying $d(x^*, x^*) = 0$ (which is $x^* = 0$).

Theorem 2.4. *Let the assumptions of Theorem 2.1 be fulfilled, except that the condition (3) is replaced by*

$$d(Tx, Ty) \leq \lambda\,d(x, y). \tag{6}$$

Then the same conclusion holds under (weaker) assumption $\lambda \in [0, 1)$.

PROOF. We will prove first that the sets of fixed points of the mappings T and T^n are the same: $Fix(T) = Fix(T^n)$ for each $n \in \mathbb{N}$. The inclusion $Fix(T) \subset Fix(T^n)$ always holds. In order to prove the converse, suppose that there exists $z \in Fix(T^n) \setminus Fix(T)$, i.e., $T^n z = z$ and $Tz \neq z$, hence $d(Tz, z) > 0$. It follows from (6) that

$$d(Tz, z) = d(T^{n+1}z, T^n z) \leq \lambda\, d(T^n z, T^{n-1}z).$$

Continuing in this way, we get

$$d(Tz, z) \leq \lambda^n d(Tz, z),$$

which is a contradiction with $\lambda^n < 1$.

Now, if $\lambda \in [0, 1/s)$, the result follows from Theorem 2.1. Suppose that $\lambda \in [1/s, 1)$, and so $\lambda^n < 1/s$ for some $n \in \mathbb{N}$. Using (6) we get that for arbitrary $x, y \in X$ with $x \perp y$,

$$d(T^n x, T^n y) \leq \lambda\, d(T^{n-1}x, T^{n-1}y) \leq \cdots \leq \lambda^n d(x, y)$$

holds, i.e., the mapping T^n satisfies condition (6) with the parameter $\lambda^n < 1/s$. Hence, by Theorem 2.1, there exists $x^* \in Fix(T^n)$. Since $Fix(T) = Fix(T^n)$, it follows that also $x^* \in Fix(T)$, which finishes the proof. □

Corollary 2.5. *The result of Theorem 2.4 remains valid in each (orthogonal) space from Table (1).*

Remark 2.6. Method of proving Banach Contraction Principle used in Theorem 2.4 is new even in b-metric spaces – in earlier papers, e.g. [**5**] or [**6**], other methods were applied or stronger assumption $\lambda \in [0, 1/s)$ was used. This result improves several earlier ones, e.g., [**7**, Theorem 4] and [**11**, Theorem 1].

Example 2.2. Let $X = [0, 1)$ and the relation $\perp$ on X be defined by $x \perp y$ if and only if $xy \leq \frac{x}{27}$, i.e., $x = 0$ or $y \leq \frac{1}{27}$. Then $(X, \perp)$ is an O-set (with orthogonal elements 0 and $\frac{1}{27}$). Define the mapping $d : X \times X \to [0, \infty)$ by

$$d(x, y) = \begin{cases} 3\max\{x, y\}, & x, y \leq \frac{1}{3}, \\ \frac{1}{3}\max\{x, y\}, & \text{otherwise}. \end{cases}$$

Then d is a b-metric-like on X with parameter $s = \frac{9}{2}$ and $(X, \perp, d, \frac{9}{2})$ is an O-complete orthogonal b-metric-like space.

Consider the mapping $T : X \to X$ given by

$$Tx = \begin{cases} \frac{1}{3}x, & x \leq \frac{1}{3}, \\ 0, & x > \frac{1}{3}. \end{cases}$$

It can be checked as in [**8**, Example 3.3] that it is an OP and OC mapping. We shall check that it satisfies condition (6) with $\lambda = \frac{1}{3}$ (note that in this case $\lambda > \frac{1}{s}$

and that with smaller λ the result cannot be obtained). Take arbitrary $x, y \in X$ with $x \perp y$, i.e., $x = 0$ or $y \leq \frac{1}{27}$. The following cases are possible.

1° $x = 0$, $0 \leq y \leq \frac{1}{3}$. Then $Tx = 0$, $Ty = \frac{1}{3}y$ and

$$d(Tx, Ty) = 3 \cdot \frac{1}{3}y = \frac{1}{3} \cdot 3y = \lambda\, d(x, y).$$

2° $x = 0$, $y > \frac{1}{3}$. Then $Tx = 0$, $Ty = 0$ and relation (6) is trivially satisfied.

3° $y \leq \frac{1}{27}$, $0 < x \leq \frac{1}{3}$. Then $Tx = \frac{1}{3}x$, $Ty = \frac{1}{3}y$ and

$$d(Tx, Ty) = 3 \cdot \frac{1}{3} \max\{x, y\} = \frac{1}{3} \cdot 3 \max\{x, y\} = \lambda\, d(x, y).$$

4° $y \leq \frac{1}{27}$, $x > \frac{1}{3}$. Then $Tx = 0$, $Ty = \frac{1}{3}y$ and

$$d(Tx, Ty) = 3 \cdot \frac{1}{3}y = y < \frac{1}{9}x = \frac{1}{3} \cdot \frac{1}{3}x = \lambda\, d(x, y).$$

Thus, in all possible cases, condition (6) is fulfilled and T satisfies all assumptions of Theorem 2.4 and, hence, it has a unique fixed point x^* satisfying $d(x^*, x^*) = 0$ (which is $x^* = 0$).

Note that if (X, d, s) is considered as a b-metric-like space, without additional $\perp$-structure, then this conclusion cannot be obtained in the same way. Namely, if one takes $x = \frac{1}{3}$ and $y = \frac{2}{3}$, then $Tx = \frac{1}{9}$, $Ty = 0$ and $d(Tx, Ty) = 3 \cdot \frac{1}{9} = \frac{1}{3}$, while $d(x, y) = \frac{1}{3} \cdot \frac{2}{3} = \frac{2}{9}$, hence no $\lambda < 1$ can be found in order that inequality $d(Tx, Ty) \leq \lambda\, d(x, y)$ holds true.

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