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Advancements in metric-like spaces with related fixed point results

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Abstract. In this paper, various fixed point results on metric-like spaces are collected. Important findings from the beginning up to the recent developments are discussed. Hence, the aim of this paper is to motivate further researches in the setting of metric-like spaces and related domains.

1. Introduction

The notion of metric space was first introduced by a French Mathematician, Maurice Frechet in 1906. After that, a lot of generalizations of metric space came into existence based on transforming the metric axioms and/or the ambient space.

The contractive mapping principle, well known as the Banach fixed point theorem (see [13]) is a widely important tool in the theory of metric spaces which assures the existence and uniqueness of fixed points of certain self-maps of metric spaces and provides a constructive method to find those fixed points. The basic idea of the contractive mapping principle has been extended in different domains (e.g., see [3, 47, 48]).

In 1994, Mathews [36] introduced the notion of partial metric space as a part of the study of denotational semantics of data flow networks and showed that the Banach contraction principle can be generalized to the partial metric context for application in program verification. In 2012, Harandi [8] reintroduced the notion of dislocated metric space as a new generalization of partial metric spaces called metric-like space. Many investigators (e.g., see [26, 31]) have established different techniques of obtaining fixed point results in metric-like spaces.

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In this survey, we will focus on highlighting the distinct and remarkable fixed point extensions in metric-like spaces in an effort to provide researchers in the area of fixed point theory with a glimpse into the advancements in fixed point theory in metric-like spaces.

2. Preliminaries

In this section, specific fundamental notations, notions and results that will be deployed subsequently are highlighted. Throughout this paper, every set X is considered non-empty, $\mathbb{N}$ is the set of natural numbers, $\mathbb{R}$ represents the set of real numbers and $\mathbb{R}_+$ the set of non-negative real numbers. We begin with the definition of partial metric space due to Matthews [36].

Definition 2.1. [36] A mapping $p : X \times X \longrightarrow \mathbb{R}_+$ is said to be a partial metric on X if for any $x, y, z \in X$, the following four conditions hold:

(P_1) $x = y$ if and only if $p(x, x) = p(y, y) = p(x, y)$;
(P_2) $p(x, x) \leq p(x, y)$;
(P_3) $p(x, y) = p(y, x)$;
(P_4) $p(x, z) \leq p(x, y) + p(y, z) - p(y, y)$.

The pair (X, p) is called a partial metric space.

A sequence $\{x_n\}$ in a partial metric space (X, p) converges to a point $x \in X$ if $\lim_{n\to\infty} p(x_n, x) = p(x, x)$. A sequence $\{x_n\}$ of elements of X is called p-Cauchy if the $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite. The partial metric space (X, p) is called complete if for each p-Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there is some $x \in X$ such that

$$\lim_{n\to\infty} p(x_n, x) = p(x, x) = \lim_{n,m\to\infty} p(x_n, x_m).$$

We then recall some definitions on a metric-like space as follows:

Definition 2.2. [8] A mapping $\sigma : X \times X \longrightarrow \mathbb{R}_+$ is said to be a metric-like on X if for any $x, y, z \in X$, the following four conditions hold:

(σ_1) $\sigma(x, y) = 0 \Rightarrow x = y$;
(σ_2) $\sigma(x, y) = \sigma(y, x)$;
(σ_3) $\sigma(x, z) \leq \sigma(x, y) + \sigma(y, z)$.

The pair (X, σ) is called a metric-like space.

Definition 2.3. [8] A sequence $\{x_n\}$ in a metric-like space (X, σ) converges to a point $x \in X$ if $\sigma(x, x) = \lim_{n\to\infty} \sigma(x_n, x)$.

Definition 2.4. [8] A sequence $\{x_n\}$ in a metric-like space (X, σ) is called σ-Cauchy sequence if the limit $\lim_{n,m\to\infty} \sigma(x_n, x_m)$ exists and is finite. The metric-like space (X, σ) is called complete if for each σ-Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there is some $x \in X$ such that

$$\lim_{n\to\infty} \sigma(x_n, x) = \lim_{n,m\to\infty} \sigma(x_n, x_m).$$

Remark 2.5. [8] Every partial metric space is a metric-like space, but the converse is not always true. An example given here under recognizes this observation.

Example 2.6. [8] Let $X = \{0, 1\}$, and let

$$\sigma(x, y) = \begin{cases} 2, & \text{if } x = y = 0; \\ 1, & \text{otherwise.} \end{cases}$$

Then (X, σ) is a metric-like space, but since $\sigma(0, 0) \nleq \sigma(0, 1)$, (X, σ) is not a partial metric space.

Remark 2.7. [8] A metric-like on X satisfies all the conditions of a metric except that $\sigma(x, x)$ may be positive for $x \in X$. Each metric-like σ on X generates a topology τ_σ on X whose base is the family of open σ-balls

$$B_\sigma(x, \epsilon) = \{y \in X : |\sigma(x, y) - \sigma(x, x)| < \epsilon\},$$

for all $x \in X$ and $\epsilon > 0$.

Definition 2.8. A mapping $T : X \longrightarrow X$ is continuous, if the following limit exists and is finite

$$\lim_{n\to\infty} \sigma(x_n, x) = \lim_{n,m\to\infty} \sigma(Tx, x).$$

Definition 2.9. [50] Let (X, σ) be a metric-like space. A sequence $\{x_n\}$ is called a $0-\sigma$-Cauchy sequence if $\lim\limits_{n\to\infty} \sigma(x_n, x_m) = 0$. The space (X, σ) is said to be $0-\sigma$-complete if every $0-\sigma$-Cauchy sequence in X converges with respect to τ_σ to a point $x \in X$ such that $\sigma(x, x) = 0$.

Definition 2.10. [45] Let $T : X \longrightarrow X$ and $\alpha : X \times X \longrightarrow \mathbb{R}^+$ be two mappings. Then, T is called α-admissible if for all $x, y \in X$ with $\alpha(x, y) \geq 1$ implies $\alpha(Tx, Ty) \geq 1$.

Definition 2.11. [21] Let (X, σ) be a metric-like space and $\alpha : X \times X \longrightarrow \mathbb{R}^+$ be a mapping. We say that an α-admissible mapping $T : X \longrightarrow X$ is α-continuous on (X, σ) if

$$x_n \to x \quad as \quad n \to \infty, \quad \alpha(x_n, x_{n+1}) \geq 1 \Rightarrow Tx_n \to Tx \ for\ all\ n \in \mathbb{N}.$$

Definition 2.12. [1] Let $f, g : X \longrightarrow X$ be two mappings and $\alpha : X \times X \longrightarrow \mathbb{R}$ be a function. We say that the pair (f, g) is α-admissible if

$$x, y \in X,\ \alpha(x, y) \geq 1 \Rightarrow \alpha(fx, gy) \geq 1 \quad and \quad \alpha(gy, fx) \geq 1.$$

Definition 2.13. [25] Let $T : X \longrightarrow X$ and $\alpha : X \times X \longrightarrow \mathbb{R}^+$. Then T is called a triangular α-admissible mapping if

(i) T is α-admissible;
(ii) $\alpha(x, z) \geq 1$ and $\alpha(z, x) \geq 1$.

Definition 2.14. [1] Let $f, g : X \longrightarrow X$ and $\alpha : X \times X \longrightarrow \mathbb{R}^+$. Then, (f, g) is called a triangular α-admissible mapping if

(i) The pair (f, g) is α-admissible;
(ii) $\alpha(x, z) \geq 1$ and $\alpha(z, x) \geq 1$ imply $\alpha(x, y) \geq 1$.

Definition 2.15. [38] Let $(X, d, \preceq)$ be a partially ordered metric space. Assume $f, g : X \longrightarrow X$ are two mappings. Then:

(i) $x, y \in X$ are said to be comparable if $x \preceq y$ or $y \preceq x$ holds;
(ii) f is said to be nondecreasing if $x \preceq y$ implies $fx \preceq fy$;
(iii) f, g are called weakly increasing if $fx \preceq gfx$ and $gx \preceq fgx$ for all $x \in X$;
(iv) f is called weakly increasing if f and I are weakly increasing, where I is denoted as the identity mapping on X.

Throughout this paper, we denote by $(X, \preceq, \sigma)$ a complete partially ordered metric-like space. The first fixed point result in metric-like space was obtained by Amini-Harandi [8]. It was shown in [8] that a self-map T on a complete metric-like space (X, σ) satisfying certain contraction conditions has a unique fixed point.

Theorem 2.1. [8] *Let (X, σ) be a complete metric-like space, and let $T : X \longrightarrow X$ be a mapping satisfying the following conditions:*

$$\sigma(Tx, Ty,) \leq \psi(M(x, y)), \tag{1}$$

for all $x, y, z \in X$ where,
$M(x, y) = \max\{\sigma(x, y), \sigma(x, Tx), \sigma(y, Ty), \sigma(x, Ty), \sigma(y, Tx), \sigma(x, x), \sigma(y, y)\}$, $\psi : \mathbb{R}^+ \to \mathbb{R}^+$ *is a nondecreasing function satisfying* $\psi(t) < t$, $\lim_{s \to t^+} \psi(s) < t$ *for all* $t > 0$ *and* $\lim_{t \to \infty}(t - \psi(t)) = \infty$. *Then T has a fixed point.*

Proof. Let $x_0 \in X$ be arbitrary, and let $x_{n+1} = Tx_n$ for $n \in \{0, 1, 2, ...\}$. Denote

$$O(x_0, n) = \{Tx_0, Tx_1, ...Tx_n\} \quad and \quad O(x_0) = \{Tx_0, Tx_1, ...Tx_n, ...\}.$$

First, we show that $O(x_0)$ is a bounded set. We shall show that for each $n \in \mathbb{N}$,

$$\delta_n(x_0) = diam(O(x_0, n)) = \sigma(Tx_0, Tx_k), \tag{2}$$

where $k = k(n) \in \{0, 1, 2, ..., n\}$. Suppose, to the contrary, that there are positive integers $1 \leq i(n) = i \leq j = j(n)$ such that

$$\delta_n(x_0) = \sigma(Tx_i, Tx_j) > 0.$$

From our assumption, we have

$$M(x_i,x_j)=\max\left\{\begin{array}{c}\sigma(x_i,x_j),\sigma(x_i,Tx_i),\sigma(x_j,Tx_j),\sigma(x_i,Tx_j),\\ \sigma(x_j,Tx_i),\sigma(x_i,x_i),\sigma(x_j,x_j)\end{array}\right\}$$
$$=\max\left\{\begin{array}{c}\sigma(x_{i-1},x_{j-1}),\sigma(x_{i-1},Tx_i),\sigma(x_{j-1},Tx_j),\sigma(x_{i-1},Tx_j),\\ \sigma(x_{j-1},Tx_i),\sigma(x_{i-1},x_{i-1}),\sigma(x_{j-1},x_{j-1})\end{array}\right\}.$$

Thus, from the above and the contractive condition on T, we have

$$\begin{aligned}\delta_n(x_0)=&\sigma(Tx_i,Tx_j)\\ &\leq\psi(\max\{\sigma(x_i,x_j),\sigma(x_i,Tx_i),\sigma(x_j,Tx_j),\sigma(x_i,Tx_j),\sigma(x_j,Tx_i),\sigma(x_i,x_i),\sigma(x_j,x_j)\})\\ &\leq\psi(\delta_n(x_0))\\ &<\delta_n(x_0),\end{aligned}$$

a contradiction. Thus, (2) holds. Since by triangle inequality,

$$\sigma(Tx_0,Tx_k)\leq\sigma(Tx_0,Tx_1)+\sigma(Tx_1,Tx_k),$$

then from (2)

$$\delta_n(x_0)\leq\sigma(Tx_0,Tx_1)+\sigma(Tx_1,Tx_k). \tag{3}$$

From our assumption on T, we have

$$\sigma(Tx_0,Tx_k)\leq\psi(M(x_1,x_k)\leq\psi(\delta_n(x_0)).$$

Now by (3),

$$\delta_n(x_0)\leq\sigma(Tx_0,Tx_1)+\psi(\delta_n(xo)).$$

Hence,

$$(I-\psi)(\delta_n(x_0))\leq\sigma(Tx_0,Tx_1),$$

where I is the identity map. Since the sequence $\{\delta_n(x_0)\}$ is nondecreasing, there exists $\lim\limits_{t\to\infty}(t-\psi(t))=\lim\limits_{n\to\infty}(\delta_n(x_0)-\psi(\delta_n(x_0)))\leq\sigma(Tx_0,Tx_1)<\infty$, a contradiction. Therefore, $\lim\limits_{n\to\infty}\delta_n(x_0)=\delta(x_0)<\infty$, that is,

$$\delta(x_0)=diam(\{Tx_0,Tx_1,...,Tx_n,...\})<\infty.$$

Now, we show that $\{x_n\}$ is a σ-Cauchy sequence. Set

$$\delta(x_n)=diam(\{Tx_n,Tx_{n+1},...\}).$$

Since $\delta(x_n)\leq\delta(x_0)$, by (2), we conclude that $\{\delta(x_n)\}$ is a nonincreasing finite nonnegative number and so it converges to some $\delta\geq 0$. We shall prove that $\delta=0$. Let $n\in\mathbb{N}$ be arbitrary, and let r,s be any positive integers such that $r,s\geq n+1$. Then $Tx_{r-1},Tx_{s-1}\in\{Tx_n,Tx_{n+1},...\}$ and hence we conclude that $M(x_r,x_s)\leq\delta(x_n)$. Then

$$\delta(Tx_r,Tx_s)\leq\psi(M(x_r,x_s)\leq\psi(\delta(x_n)).$$

Hence, we get

$$\delta(x_{n+1}) = \sup\{\sigma(Tx_r, Tx_s) : r, s \geq n+1\} \leq \psi(\delta(x_n)).$$

Therefore, as $\delta \leq \delta(x_n)$ for all $n \geq 0, \delta \leq \psi(\delta(x_n))$. Suppose that $\delta > 0$. Then we get

$$\delta \leq \lim_{n\to\infty} \psi(\delta(x_n)) = \lim_{s\to\delta^+} \psi(s) < \delta,$$

a contradiction. Therefore, $\delta = 0$. Thus, we have proved that

$$\lim_{n\to\infty} diam(\{Tx_n, Tx_{n+1}, ...\}) = 0.$$

Hence, from the triangle inequality, we conclude that $\{x_{n+1} = Tx_n\}$ is a σ-Cauchy sequence. By the completeness of X, there is some $u \in X$ such that $\lim_{n\to\infty} Tx_n = u$, that is,

$$\lim_{n\to\infty} \sigma(Tx_n, u) = \sigma(u, u) = \lim_{m,n\to\infty} \sigma(Tx_n, Tx_m) = 0.$$

We show that $Tu = u$. Suppose, by the way of contradiction, that $\sigma(Tu, u) > 0$. Then we have

$$\begin{aligned} \sigma(Tu, u) &\leq \sigma(u, Tx_{n+1}) + \sigma(Tu, Tx_{n+1}) \\ &\leq \sigma(u, Tx_{n+1}) + \psi(M(u, x_{n+1})), \end{aligned} \tag{4}$$

where,

$$\begin{aligned} M(u, x_{n+1}) &= \max \left\{ \begin{array}{c} \sigma(u, x_{n+1}), \sigma(u, Tu), \sigma(x_{n+1}, Tx_{n+1}), \\ \sigma(u, Tx_{n+1}), \sigma(x_{n+1}, Tu), \sigma(u, u), \\ \sigma(x_{n+1}, x_{n+1}) \end{array} \right\} \\ &= \max \left\{ \begin{array}{c} \sigma(u, Tx_n), \sigma(u, Tu), \sigma(Tx_n, Tx_{n+1}), \\ \sigma(u, Tx_{n+1}), \sigma(Tx_n, Tu), \sigma(u, u), \\ \sigma(Tx_n, Tx_n) \end{array} \right\}. \end{aligned}$$

From the triangle inequality, we have

$$|\sigma(Tu, Tx_{n+1}) - \sigma(Tu, u)| \leq \sigma(u, Tx_{n+1}) \to 0 \quad as \quad n \to \infty.$$

Thus, $\lim_{n\to\infty} \sigma(Tu, Tx_{n+1}) = \sigma(Tu, u)$. Since $\lim_{n\to\infty} \sigma(u, Tx_n) = 0$, $\lim_{n\to\infty} \sigma(Tx_n, Tu) = \sigma(Tu, u)$, for large enough n, we have

$$M(u, x_{n+1}) = \max\{\sigma(u, Tu), \sigma(Tx_n, Tu)\}.$$

If $M(u, x_{n+1}) = \sigma(u, Tu)$, then from (4), we get

$$\sigma(Tu, u) \leq \sigma(u, Tx_{n+1}) + \psi(\sigma(Tu, u)).$$

Letting n tends to infinity, we get

$$0 < \sigma(Tu, u) \leq \psi(\sigma(Tu, u)) < \sigma(Tu, u),$$

a contradiction. If $M(u, x_{n+1}) = \sigma(Tx_n, Tu)$, then we have

$$\sigma(Tx_n, Tu) = M(u, x_{n+1}) \geq \sigma(Tu, u),$$

and so $\sigma(Tx_n, Tu) \to \sigma(Tu, u)^+$. Then from (4) and our assumptions on ψ, we get $\sigma(Tu, u) < \sigma(Tu, u)$, a contradiction. Thus, $\sigma(Tu, u) = 0$ and so $Tu = u$. □

Theorem 2.2. [8] *Let (X, σ) be a complete metric-like space, and let $T : X \longrightarrow X$ be a map such that*

$$\sigma(Tx, Ty,) \leq \sigma(x, y) - \varphi(\sigma(x, y)), \tag{5}$$

for all $x, y, z \in X$, where $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is a nondecreasing continuous function such that $\varphi(t) = 0$ if and only if $t = 0$. Then T has a unique fixed point.

Proof. Let $x_0 \in X$ and define $x_{n+1} = Tx_n$ for $n \geq 0$. Then by assumption,

$$\sigma(x_{n+1}, x_{n+2}) = \sigma(Tx_n, Tx_{n+1}) \leq \sigma(x_n, x_{n+1}) - \varphi(\sigma(x_n, x_{n+1})), \tag{6}$$

for each $n \in \mathbb{N}$. Then $\{\sigma(x_n, x_{n+1})\}$ is a nonnegative nonincreasing sequence and hence possesses a limit $r_0 \leq 0$. Since φ is nondecreasing, then from (6), we get

$$\sigma(x_{n+1}, x_{n+2}) \leq \sigma(x_n, x_{n+1}) - \varphi(r_0)$$

for each $n \in \mathbb{N}$. Then $r_0 \leq r_0 - \varphi(r_0)$ and so $r_0 = 0$. Therefore,

$$\lim_{n\to\infty} \sigma(x_n, x_{n+1}) = 0.$$

Now, we show that $\{x_n\}$ is a Cauchy sequence. $Fix\varepsilon > 0$ and choose N such that

$$\sigma(x_n, x_{n+1}) < \min\left\{\frac{\varepsilon}{2}, \varphi(\frac{\varepsilon}{2})\right\} \quad for \quad n \geq N.$$

We show that if $\sigma(x, x_N) \leq \varepsilon$, then $\sigma(Tx, x_N) \leq \varepsilon$. To show the claim, let us assume first that $\sigma(x, x_N) \leq \frac{\varepsilon}{2}$. Then

$$\begin{aligned}\sigma(Tx, x_N) \leq& \sigma(Tx, Tx_N) + \sigma(Tx_N, x_N)\\ \leq& \sigma(x, x_N) - \varphi(\sigma(x, x_N)) + \sigma(x_{N+1}, x_N)\\ <& \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon.\end{aligned}$$

Now we assume that $\frac{\varepsilon}{2} < \sigma(x, x_N) \leq \varepsilon$. Then $\varphi(\sigma(x, x_N)) \geq \varphi(\frac{\varepsilon}{2})$. Therefore, from the above, we have

$$\begin{aligned}\sigma(Tx, x_N) &\leq \sigma(Tx, Tx_N) + \sigma(Tx_N, x_N)\\ &\leq \sigma(x, x_N) - \varphi\left(\frac{\varepsilon}{2}\right) + \varphi\left(\frac{\varepsilon}{2}\right)\\ &= \sigma(x, x_N) \leq \varepsilon.\end{aligned}$$

Since $\sigma(x_{N+1}, x_N) \leq \varepsilon$, from the above, we deduce that $\sigma(x_n, x_N) \leq \varepsilon$ for each $n \geq N$. Since $\varepsilon > 0$ is arbitrary, we get $\lim_{m,n\to\infty} \sigma(x_m, x_n) = 0$ and so $\{x_n\}$ is a

Cauchy sequence. Since X is complete, there is some $u \in X$ such that $\lim_{n\to\infty} x_n = u$, that is

$$\lim_{n\to\infty} x_n = \sigma(u,u) = \lim_{m,n\to\infty} \sigma(Tx_n, Tx_m) = 0. \tag{7}$$

Since

$$\sigma(x_{n+1}, Tu) = \sigma(Tx_n, Tu) \leq \sigma(x_n, u) - \varphi(\sigma(x_n, u)) \tag{8}$$

and φ is continuous, from (7) and (8), we have

$$\lim_{n\to\infty} \sigma(x_n, Tu) = 0. \tag{9}$$

Since

$$\sigma(u, Tu) \leq \sigma(x_n, u) + \sigma(x_n, Tu),$$

by (7) and (9), we infer that $\sigma(u, Tu) = 0$ and so $Tu = u$. To prove the uniqueness, let v be another fixed point of T, that is, $Tv = v$. Then

$$\sigma(u,v) = \sigma(Tu, Tv) \leq \sigma(u,v) - \varphi(\sigma(u,v)),$$

which gives $\varphi(\sigma(u,v) = 0)$ and so $u = v$. □

3. Sequent of Amini-Harandi Results

In this section, important extensions of the results of Amini-Harandi [8] are discussed. One of the earliest generalizations of Amini-Harandi's result was given by Isik and Turkoglu [22]. We first consider this result.

3.1. Isik and Turkoglu (2013). Isik and Turkoglu [22] established some fixed point theorems for weakly contractive mappings defined in odered metric-like spaces. They ([22]) proved some new fixed points results in ordered partial metric spaces.

Theorem 3.1. [22] *Let $(X, \preceq, \sigma)$ be a complete partially ordered metric-like space. Let $T : X \longrightarrow X$ be a continuous and non-decreasing mapping such that for all comparable $x, y \in X$,*

$$\psi(\sigma(Tx, Ty) \leq \psi(M(x,y)) - \phi(M(x,y)),$$

where

$$M(x,y) = \max\{\sigma(x,y), \sigma(x,Tx), \sigma(x,Tx), \sigma(x,x), \sigma(y,y), [\sigma(x,Ty) + \sigma(Tx,y)/2\}$$

and

(i) *$\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous monotone nondecreasing function with $\psi(t) = 0$ if and only if $t = 0$;*
(ii) *$\psi : \mathbb{R}^+ \to \mathbb{R}^+$ is a lower semi-continuous function with $\phi(t) = 0$ if and only if $t = 0$.*

If there exists $x_0 \in X$ with $x_0 \leq Tx_0$, then T has a fixed point.

3.2. Shukla and Fisher (2013). Shukla and Fisher [51] defined Prešic-type mappings and proved some common fixed point theorems for Prešic-type mappings in metric-like space.

Definition 3.1. [51] Let (X, σ) be a metric-like space, k a positive integer and $f : X^k \longrightarrow X$ a mapping. f is said to be Prešic-type if

$$\sigma(f(x_1, x_2, ..., x_k), f(x_2, x_3, ..., x_{k+1})) \leq \Sigma_{i=1}^{k} \alpha_i \sigma(x_i, x_{i+1})$$

for all $x_1, x_2, ..., x_{k+1} \in X$, where α_i are non-negative constants such that $\Sigma_{i=1}^{k} \alpha_i < 1$.

Their main result is the following.

Theorem 3.2. [51] *Let (X, σ) be a metric-like space, k a positive integer and $f : X^k \longrightarrow X$, $g : X \longrightarrow X$ be two mappings such that $f(x^k) \subset g(x)$ and $g(x)$ is a complete subspace of X. Suppose that the following condition holds:*

$$\sigma(f(x_1, x_2, ..., x_k), f(x_2, x_3, ..., x_{k+1})) \leq \Sigma_{i=1}^{k} \alpha_i \sigma(gx_i, gx_{i+1})$$

for every $x_1, x_2, ..., x_{k+1} \in X$, where α_i are non-negative constants such that $\alpha_1, \alpha_1, ..., \alpha_k \leq 1$. Then f and g have a unique point of coincidence $v \in X$ and $\sigma(v, v) = 0$. Moreover, if f and g are weakly compatible, then v is the unique common fixed point of f and g.

3.3. Shukla, Radenovic and Rajic (2013). Shukla et al. [53] introduced the notion of $0-\sigma$-complete metric-like space and proved some common fixed point theorems in such spaces. The main result of Shukla, Radenovic and Rajic [53] which is a generalization and improvement of Theorem 2.1[8], is the following:

Theorem 3.3. [53] *Let (X, σ) be a metric-like space. Suppose the mappings $f, g : X \longrightarrow X$ satisfies*

$$\sigma(fx, fy) \leq \psi(M(x, y))$$

for all $x, y \in X$, where $\psi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a nondecreasing function satisfying $\psi_t < t$ for all $t > 0$

$$\lim_{s \to t} \psi(s) < t \quad for \quad all \quad t > 0, \lim_{s \to \infty} (t - \psi(t)) = \infty$$

and

$$M(x, y) = \max\{\sigma(gx, gy), \sigma(gx, fx), \sigma(gy, fy), \sigma(gx, fy), \sigma(gy, fx), \sigma(gx, gx), \sigma(gy, gy)\}.$$

If the range of g contains the range of f and $f(X)$ or $g(X)$ is a closed subset of X, then f and g have a unique point of coincidence in X. Moreover, if f and g are weakly compatible, then f and g have a unique fixed point x and $\sigma(x, x) = 0$

Theorem 3.4. [53] *Let (X, σ) be a $0 - \sigma$-complete metric-like space. Suppose the mappings $f, g : X \longrightarrow X$ satisfy*

$$\psi(\sigma(fx, fy)) \leq \psi(\sigma(gx, gy)) - \varphi(\sigma(gx, gy))$$

for all $x, y \in X$, where $\psi \in \Psi$, $\varphi \in \Phi$. If the range of g contains the range of f and $f(X)$ or $g(X)$ is a closed subset of X, then f and g have a unique fixed point x and $\sigma(x,x) = 0$. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point v and $\sigma(x,x) = 0$.

3.4. Malhotra, Radenovic and Shukla (2013). Malhotra et al. [32] obtained the fixed point results for F-type contractions which satisfy weaker conditions than the monotonicity of self-mapping of a partially ordered metric-like space. And they also proved fixed point result for F-expansive mappings. The main result of Malhotra et al. [32] is the following.

Theorem 3.5. [32] *Let $(X, \sigma, \sqsubseteq)$ be a partially ordered metric-like space and let $f, g : X \longrightarrow X$ be a mapping such that $f(X) \subset g(X)$ and $g(X)$ is σ-complete satisfying the following conditions:*

(i) *if $x, y \in X$ such that $g(x) \asymp f(x) = gy$, then $f(x) \asymp f(y)$;*
(ii) *there exists $x_0 \in X$ such that $gx_0 \asymp fx_0$;*
(iii) *there exists $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $gx \asymp gy$, we have*

$$\sigma(fx, fy) > 0 \Rightarrow \tau + F(\sigma(fx, fy)) \leq F(\max\{\sigma(gx, gy), \sigma(gx, fx), \sigma(gy, fy)\});$$

(iv) *if $\{x_n\}$ is a sequence in (X, σ) converging to $x \in X$ and $\{x_n : n \in \mathbb{N}\}$ is well ordered, then $x_n \asymp x$ for sufficiently large n.*

Suppose F is continuous,then the pair (f, g) have a point of coincidence $v \in V$ and $\sigma(v, v) = 0$. Furthermore, if the set of coincidence points of the pair (f, g) is g-well ordered then the pair (f, g) have a unique point of coincidence. If in addition, the pair (f, g) is weakly compatible, then there exists a unique common fixed point of the pair (f, g).

Theorem 3.6. [32] *Let $(X, \sigma, \sqsubseteq)$ be a partially ordered metric-like space and let $f : g : X \longrightarrow X$ be a mappings such that $f(X) \supset g(X)$ and $g(X)$ is $\sigma -$ complete. Suppose that the following hold:*

(i) *if $x, y \in X$ such that $fx \asymp gx = fy$, then $gx \asymp gy$;*
(ii) *there exists $x_0 \in X$ such that $fx_0 \asymp gx_0$;*
(iii) *there exist $F \in \mathcal{F}$ and $\tau > 0$ such that for all $x, y \in X$ satisfying $gx \asymp gy$, we have*

$$\sigma(gx, gy) > 0 \Rightarrow F(\sigma(fx, fy)) \geq F(\sigma(fx, fy)) \geq F(\sigma(gx, gy)) + \tau;$$

(iv) *if $\{x_n\}$ is a sequence in (X, σ) converging to $x \in FX$ and $\{x_n : n \in\}$ is well ordered,then $x_n \asymp x$ for sufficiently large n.*

Suppose F is continuous, then the pair (f, g) have a point of coincidence $v \in X$ and $\sigma(v, v) = 0$. Furthermore, if the set of coincidence points of the pair (f, g) is g-well ordered, then the pair(f, g) have a unique point of coincidence. If in addition, *the pair (f, g) is weakly compatible, then there exists a unique common fixed point of the pair (f, g).*

3.5. Shobkolaei, Sedghi, Roshan and Hussain (2013). Shobkolaei et al. [49] demonstrated a fundamental lemma for the convergence of sequences in metric-like spaces, and proved some Suzuki-type fixed point results in the setup of metric-like spaces.

Theorem 3.7. [49] *Let (X, σ) be a complete metric-like space. Let $T : X \longrightarrow X$ be a self-map and let $\theta : [0, 1) \to (\frac{1}{2}, 1]$ be defined by if there exists $r \in [0, 1)$ such that for each $x, y \in X$*

$$\theta(r)\sigma(x, Tx) \leq \sigma(x, y) \Rightarrow \sigma(Tx, Ty) \leq r\sigma(x, y).$$

Then T has a unique fixed point $z \in X$ and for each $x \in X$, the sequence $\{T^n x\}$ converges to z.

Theorem 3.8. [49] *Let (X, σ) be a complete metric-like space. Let $f, g : X \longrightarrow X$ be two self-mappings. Suppose that there exists $r \in [0, 1)$ such that*

$$\max\{\sigma(f(x), gf(x)), \sigma(g(x), fg(x))\} \leq r \min\{\sigma(x, f(x)), \sigma(x, g(x))\}$$

for every $x \in X$ and that

$$\alpha(y) = \inf\{\sigma(x, y) + \min\{\sigma(x, S(x)), \sigma(x, T(x))\} : x \in X\} > 0$$

for every $y \in X$ with y that is not a common fixed point of f and g. Then there exists $z \in X$ such that $z = f(z) = g(z)$. Moreover, if $v = f(v) = g(v)$, then $\sigma(v, v) = 0$.

3.6. Al-Mezel, Chen, Karapinar and Rakocevic (2014). Al-Mezel et al. [4] established some fixed point theorems for α-admissible mappings in the context of metric-like space via various auxiliary functions. In Particular, they proved the existence of a fixed point of the generalized Meir-Keeler type $\alpha - \phi$-contractive self-mapping T defined on a metric-like space X, and unify several fixed point theorems for the generalized cyclic contractive mappings.

Definition 3.2. [30] A mapping $T : A \cup B \longrightarrow A \cup B$ is called cyclic if

$$T(A) \subset B \quad \textit{and} \quad T(B) \subset A.$$

Theorem 3.9. [4] *Let A and B be two nonempty closed subsets of a complete metric-like space (X, d) and suppose $T : A \cup B \longrightarrow A \cup B$ satisfies the following:*

(i) T is a cyclic map;

(ii) $d(Tx, Ty) \leq k.d(x, y)$ for all $x \in A, y \in B$ and $k \in (0, 1)$.

Then $A \cap B$ is nonempty and T has a unique fixed point in $A \cap B$.

Definition 3.3. [4]. A function $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ is said to be a Meir-keeler type mapping [37], if for each $\eta \in [0, \infty)$, there exists $\delta > 0$ such that, for $t \in \mathbb{R}^+$ with $\eta \leq t < \eta + \delta$, we have $\gamma(t) < \eta$.

Let Φ be the class of all function $\phi : \mathbb{R}_5^+ \to \mathbb{R}^+$ satisfying the following conditions:

(ϕ_1) ϕ is an increasing and continuous function in each coordinate;

(ϕ_2) for $t > 0$ $\phi(t,t,t,2t,2t) < t, \phi(t,0,0,t,t) < t$ and $\phi(0,0,t,t,0) < t$;

(ϕ_3) $\phi(t_1,t_2,t_3,t_4,t_5) = 0$ if and only if $t_1 = t_2 = t_3 = t_4 = t_5 = 0$.

Definition 3.4. [7] Let (X,σ) be a metric-like space and let $\alpha : X \times X \to \mathbb{R}^+$. One says that $T : X \longrightarrow X$ is called a generalized Meir-Keeler type $\alpha-\phi$-contractive mapping if for each $\eta > 0$ there exist $\delta > 0$ such that

$$\begin{aligned}\eta &\leq \phi(\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty),\sigma(x,Ty),\sigma(y,Tx))\\ &< \eta+\delta \longrightarrow \alpha(x,y)\sigma(Tx,Ty) < \eta\end{aligned}$$

for all $x,y \in X$ and $\phi \in \Phi$

Remark 3.5. [9] Note that if T is a generalized Meir-Keeler type $\alpha-\phi$-contractive mapping. Then we have, for all $x,y \in X$ and $\phi \in \Phi$,

$$\alpha(x,y)\sigma(Tx,Ty) \leq \phi(\sigma(x,y),\sigma(Tx,Ty),\sigma(y,Ty),\sigma(x,Ty),\sigma(y,Tx)).$$

Fixed Point Theorem via the α-Admissible Meir-Keeler-Type-Mappings.

Theorem 3.10. [4] *Let (X,σ) be a complete metric-like space and let $T : X \longrightarrow X$ be a generalized Meir-Keeler type $(\alpha-\phi)$-contractive mapping, where α is transitive. Suppose that*

(i) T is α-admissible;

(ii) there exist $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$;

(iii) T is continuous.

Then there exists $u \in X$ such that $Tu = u$.

Fixed point theorem via auxiliary functions.

Definition 3.6. [4] Let (X,σ) be a metric-like space and let $\alpha : X \times X \longrightarrow \mathbb{R}^+$. One says that T is called a generalized $(\varphi,\phi,\psi,\xi)-\alpha$-contractive mapping if T is α-admissible and satisfies the following inequality:

$$\begin{aligned}\alpha(x,y)\varphi(\sigma(Tx,Ty)) \leq \phi(\psi(\sigma(x,y))),\psi(\sigma(x,Tx)),\psi(\sigma(y,Ty)),\psi(\sigma(x,Ty)),\psi(\sigma(y,Tx))\\ -\xi\left(\max\left\{\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty),\frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\}\right)\end{aligned}$$

for all $x,y \in X$, where $\psi \in \Psi$, $\phi \in \Phi$, $\varphi \in \Theta$ and $\xi \in \Xi$.

Theorem 3.11. [4] *Let (X,σ) be a complete metric-like space and let $T : X \longrightarrow X$ be a $(\varphi,\phi,\psi,\xi)-\alpha$-contractive mapping , where α is transitive. Suppose that*

(i) T is α-admissible;

(ii) there exists $x_0 \in X$ such that $\alpha(x_0,Tx_0) \geq 1$;

(iii) T is continuous.

Then there exists $x \in X$ such that $Tx = x$.

Fixed Point Theorems via the Weaker Meir-Keeler Function φ

Definition 3.7. [15] One calls $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ a weaker Meir-Keeler function if, for each $\eta > 0$, there exists $\delta > 0$ such that, for $t \in [0, \infty)$ with $\eta \leq t < \eta + \delta$, there exists $n_0 \in \mathbb{N}$ such that $\varphi^{n0}(t) < \eta$. One denotes by $\mathcal{M}$ the class of nondecreasing functions $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ satisfying the following conditions:

(φ_1) $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a weaker Meir-Keeler function;
(φ_2) $\varphi(t) > 0$ for $t > 0$ and $\varphi(0) = 0$;
(φ_3) for all $t > 0$, $\{\varphi^n(t)_{n\in\mathbb{N}}$ is decreasing$\}$;
(φ_4) if $\lim\limits_{n\to\infty} t_n = \gamma$, then $\lim\limits_{n\to\infty} \varphi(t_n) \leq \gamma$.

Definition 3.8. [15] Let (X, σ) be a metric-like space and let $\alpha : X \times X \longrightarrow \mathbb{R}^+$. One says that $T : X \to X$ is called a generalized weaker Meir-Keeler type $\alpha - (\mu, \varphi)$-contractive mapping if T is α-admissible and satisfies

$$\alpha(x, y)\sigma(Tx, Ty) \leq \mu(M(x, y)) - \varphi(M(x, y)),$$

for all $x, y \in X$, where $\mu \in \mathcal{M}$, $\varphi \in \Theta$ and

$$M(x, y) = \max\left\{\sigma(x, y), \sigma(x, fx), \sigma(y, fy), \frac{\sigma(x, fy) + \sigma(y, fx)}{4}\right\}.$$

Theorem 3.12. [4] *Let (X, σ) be a complete metric-like space and let $T : X \longrightarrow X$ be a generalized weaker Meir-Keeler type $\alpha - (\mu, \varphi)$-contractive mapping , where α is transitive. Suppose that*

(i) T is α-admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;
(iii) T is continuous.

Then there exists $x \in X$ such that $Tx = x$.

3.7. Fadali, Ahmad, Rakocevic and Rajovic (2015). Fadali et al. [19] used the contex of $0 - \sigma$-complete metric-like space and obtained some common fixed points of maps that satisfy the generalized so-called (F, ψ, φ)-weak contractive condition.

Theorem 3.13. [19] *Let (X, σ) be a $0 - \sigma$-complete metric-like space. Suppose that the mappings $f, g : X \longrightarrow X$ satisfy*

$$\psi(\sigma(fx, fy)) \leq F(\psi(\sigma(gx, gy)), \varphi(\sigma(gx, gy)))$$

for all $x, y \in X$ where $\psi \in \Psi, (\varphi \in \Phi)$ and $F \in C$. If the range of g contains the range of f and $f(X)$ or $g(X)$ is a closed subsets of X, then f and g have a unique point of coincidence in X. Moreover, if f and g are weakly compatible, then f and g have a unique common fixed point $x \in X$ and $\sigma(x, x)$=0 =$\sigma(fx, fx)$= $\sigma(gx, gx)$.

Theorem 3.14. [19] *Let (X, σ) be a $0-\sigma$-complete metric-like space and $f, g : X \longrightarrow X$ be two mappings such that for some $\psi \in \Psi, (\varphi \in \Phi)$, $F \in C$ and $x, y \in X$, there exists*

$$u(x,y) \in \left\{\sigma(x,y), \sigma(x,fx), \sigma(y,gy), \frac{1}{4}\sigma(x,gy) + \sigma(y,fx)\right\},$$

such that

$$\psi(\sigma(fx,gy)) \le F(\psi(u(x,y)), \varphi(u(x,y)).$$

Then f and g have a unique common fixed point.

3.8. Aydi and Karapinar (2015). Aydi and Karapinar [10] introduced the concept of generalized $\alpha-\psi$ contraction in the context of metric-like spaces and established some related fixed point theorems.

Definition 3.9. [45] For a nonempty set X, let $T : X \longrightarrow X$ and $\alpha : X \times X \longrightarrow [0, \infty)$ be given mappings. We say that T is α-admissible if for all $x, y \in X$, we have

$$\alpha(x,y) \ge 1 \Rightarrow \alpha(Tx,Ty) \ge 1$$

Definition 3.10. [45] Let (X, d) be a metric space and $T : X \longrightarrow X$ be a given mapping. We say that T is an $\alpha-\psi$ contractive mapping if there exist two functions $\alpha : X \times X \to \mathbb{R}^+$ and $\psi \in \Psi$ such that

$$\alpha(x,y)\ d(Tx,Ty) \le \psi\ (d(x,y))$$

for all $x, y \in X$.

Definition 3.11. [10] Let (X, σ) be a metric-like space and $T : X \longrightarrow X$ be given mappings. We say that T is a generalized $\alpha-\psi$-contractive mappings of type A if there exist two functions $\alpha : X \times X \longrightarrow \mathbb{R}^+$ and $\psi \in \Psi$ such that

$$\alpha\ (x,y)\sigma(Tx,Ty) \le \psi(M(x,y))$$

for all $x, y \in X$, where

$$M(x,y) = \max\left\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty), \frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\}.$$

Theorem 3.15. [10] *Let (X, σ) be a complete metric-like space and $T : X \longrightarrow X$ be a generalized $\alpha-\psi$ contractive mapping of type A. Suppose*

(i) *T is α-admissible;*
(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \ge 1$;*
(iii) *T is continuous.*

Then there exists $x \in X$ such that $\sigma(x, x) = 0$. Assume in addition that

(H_1) *if $\sigma(x,x)$=0 for some $x \in X$, then $\alpha(x, x) \ge 1$ and such x is a fixed point of T.*

Definition 3.12. [10] Let (X, σ) be a metric-like space and $T : X \longrightarrow X$ be a given mapping. We say that T is a generalized $\alpha - \psi$ contractive mapping of type B if there exist two functions $\alpha : X \times X \to \mathbb{R}^+$ and $\psi \in \Psi$ such that

$$\alpha(x, y)\sigma(Tx, Ty) \leq \psi(M(x, y))$$

for all $x, y \in X$, where

$$M(x, y) = \max\{\sigma(x, y)\sigma(x, Tx), \sigma(y, Ty)\}.$$

Theorem 3.16. [10] *Let (X, σ) be a complete metric-like space and $T : X \longrightarrow X$ be a generalized $\alpha - \psi$ contractive mapping of type B. Suppose that*

(i) T is α-admissible;
(ii) there exist $x_0 \subset X$ such that $\alpha(x_0, Tx_0) \geq 1$;
(iii) T is continuous.

Then there exists $u \in X$ such that $\sigma(x, x) = 0$. If in addition (H_1) in Theorem 3.15 holds, then x is a fixed point of T, that is $Tx = x$.

Definition 3.13. [10] Let (X, σ) be a metric-like space and $T : X \longrightarrow X$ be a given mapping. We say that T is an $\alpha - \psi$ contractive mapping if there exist two functions $\alpha : X \times X \to \mathbb{R}^+$ and $\psi \in \Psi$ such that

$$\alpha(x, y)\sigma(Tx, Ty) \leq \psi(\sigma(x, y)),$$

for all $x, y \in X$.

Theorem 3.17. [10] *Let (X, d) be a complete metric-like space and $T : X \longrightarrow X$ be an $\alpha - \psi$ contractive mapping. Suppose that*

(i) T is α-admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;
(iii) T is continuous.

Then there exist $x \in X$ such that $\sigma(x, x) = 0$. If in addition, (H_1) in Theorem 3.15 holds, then x is a fixed point of T.

3.9. Alsulami, Karapinar and Piri (2015). Alsulami et al. [6] introduced the notion of modified F-contractive mapping in the setting of complete metric-like spaces and investigated the existence and uniqueness of fixed point of such mappings.

Definition 3.14. [11] Let (X, σ) be a metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be modified F-contraction of type 1 if there exists $\tau > 0$ such that

$$\begin{aligned}\frac{1}{2}\sigma(x, Tx) < \sigma(x, y) \Rightarrow \tau + F(\sigma(Tx, Ty))\\ \leq \alpha F(\sigma(x, y)) + \beta F(\sigma(x, Tx)) + \gamma F(\sigma(y, Ty)),\end{aligned}$$

for all $x, y \in X$ with $x \neq y$, where $\gamma \in [0,1)$ and $\alpha, \beta \in [0,1]$ are real numbers such that $\alpha + \beta + \gamma = 1$ and $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a mapping satisfying the following conditions:

(F_1) F is strictly increasing; that is, for all $\alpha, \beta \in \mathbb{R}^+$ such that $F(\alpha) < F(\beta)$;
(F_2) for any sequence $\{\alpha_n\}_{n=1}^{\infty}$ of positive real numbers, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to\infty} F(\alpha_n) = -\infty$.

Theorem 3.18. [6] *Let (X, σ) be a complete metric-like space and T a modified F-contraction of type I. Then, T has a fixed point $x \in X$.*

Definition 3.15. [55] Let (X, σ) be a metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be modified F-contraction of type III if there exists $\tau > 0$ such that

$$\sigma(Tx, Ty) > 0 \Rightarrow \tau + F(\sigma(Tx, Ty)) < \alpha F(\sigma(x, y)) + \beta F(\sigma(x, Tx)) + \gamma F(\sigma(y, Ty)),$$

for all $x, y \in X$ with $x \neq y$ where $\gamma \in [0,1)$ and $\alpha, \beta \in [0,1]$ are real numbers such that $\alpha + \beta + \gamma = 1$ and $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a mapping satisfying the conditions (F_1) and (F_2) introduced in Definition 3.14

Theorem 3.19. [6] *Let (X, σ) be a complete metric-like space and T is a continuous modified F-contraction of type III. If $\sigma(Tx, Tx) \leq \sigma(x, x)$ for all $x \in X$, then T has a fixed point $x \in X$.*

Definition 3.16. [6] Let (X, σ) be a metric-like space. A self-mapping $T : X \longrightarrow X$ is said to be modified F-contraction of type IV if there exists $\tau > 0$ such that

$$\sigma(Tx, Ty) > 0 \Rightarrow \tau + F(\sigma(Tx, Ty)) < F(\sigma(x, y)),$$

for all $x, y \in X$ with $x \neq y$ where $F : \mathbb{R}^+ \longrightarrow \mathbb{R}$ is a mapping satisfying the condition ($F1$) and ($F2$) introduced in Definition 3.14

Theorem 3.20. [6] *Let (X, σ) be a complete metric-like space and T is a continuous modified F-contraction of type IV. If $\sigma(Tx, Tx) \leq \sigma(x, x)$ for all $x \in X$, then T has a fixed point $x \in X$.*

3.10. Karapinar, Kutbi, Piri and Ragan (2015). Karapinar et al. [27] introduced the notion of conditionally F-contraction in the setting of complete metric-like spaces and investigated the existence of fixed points of such mappings.

Definition 3.17. [27] Let (X, σ) be a metric-like space. A mapping $T : X \longrightarrow X$ is said to be a conditionally F-contraction of type (A) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $\sigma(Tx, Ty) > 0$,

$$\frac{1}{2}\sigma(x, Tx) < \sigma(x, y) \Rightarrow \tau + F(\sigma(Tx, Ty)) \leq F(M_T(x, y)),$$

where

$$M_T(x,y) = \max\left\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty), \frac{\sigma(x,Ty)+\sigma(y,Tx)}{4}\right\}.$$

Theorem 3.21. [27] *Let (X,σ) be a complete metric-like space. If T is a conditionally F-contraction of type (A), then T has a fixed point $x \in X$.*

Definition 3.18. [27] Let (X,σ) be a metric-like space. A mapping $T : X \longrightarrow X$ is said to be a conditionally F-contraction of type (B) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $\sigma(Tx,Ty) > 0$,

$$\frac{1}{2}\sigma(x,Tx) < \sigma(x,y) \Rightarrow \tau + F(\sigma(Tx,Ty)) \leq F(\max\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty)\}).$$

Definition 3.19. [27] Let (X,σ) be a metric-like space. A mapping $T : X \longrightarrow X$ is said to be a conditionally F-contraction of type (C) if there exist $F \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $\sigma(Tx,Ty) > 0$,

$$\frac{1}{2}\sigma(x,Tx) < \sigma(x,y) \Rightarrow \tau + F(\sigma(Tx,Ty)) \leq F(\sigma(x,y)).$$

Theorem 3.22. [27] *Let (X,σ) be a complete metric-like space. If T is a conditionally F-contraction of type (B), then T has a fixed point $x \in X$.*

Theorem 3.23. [27] *Let (X,σ) be a complete metric-like space. If T is a conditionally F-contraction of type (C), then T has a fixed point $x \in X$.*

3.11. Shukla and Nashine (2016). Shukla and Nashine [54] defined the cyclic-Presic-Ciric operators in metric-like spaces and proved some fixed point results for such operators.

Definition 3.20. [30] Let $T : A \cup B \longrightarrow A \cup B$ be a class of mappings satisfying the following conditions:

(*i*) $T(A) \subseteq B$ and $T(B) \subseteq A$;
(*ii*) $d(Tx,Ty) \leq \lambda d(x,y)$ for all $x \in A$ and $y \in B$, where $\lambda \in [0,\infty)$.

The mapping satisfying the above conditions is called cyclic contractions.

Definition 3.21. [54] Let X be a nonempty set and $A_1, A_2, ...A_m$ be nonempty subsets of X. A sequence$\{x_n\}_{n\in\mathbb{N}}$ in X is called m-cyclic sequence if:

(*i*) there exists $i \in \{1, 2, ...m\}$such that $x_1 \in A_i$;
(*ii*) $x_n \in A_i$ for some $n \in \mathbb{N}, i \in \{1, 2, ..., m\}$ implies that $x_{n+1} \in A_{i+1}$, where $A_{m+j} = A_j$ for all $j \in \mathbb{N}$.

The main result of Shukla et al [54] is the following.

Theorem 3.24. [54] *Let $A_1, A_2, ..., A_m$ be closed subsets of a $0-\sigma$ complete metric-like space (X,σ), k a positive integer and $X = \bigcup_{i=1}^m A_i$. Let $f : X^k \longrightarrow X$ be a cyclic-Presic-Ciric operator. Then $\bigcap_{i=1}^m A_i \neq 0$ and f has a fixed point $x \in \bigcap_{i=1}^m A_i$*

, such that $\sigma(x,x)=0$. Moreover, if $i\in\{1,2,...m\}$ and $x_i\in A_1, A_2\in A_{i+1},...,\in A_{i+k-1}$ be arbitrary points, then the sequence $\{x_n\}$ defined by

$$x_{n+k}=f(x_n,(x_{n+1},...,(x_{n+k-1})$$

for all $n\in\mathbb{N}$ is an m-cyclic sequence and converges to a fixed point of f.

Theorem 3.25. [54] *Let $A_1,A_2,...,A_m$ be closed subsets of a $0-\sigma$ complete metric-like space (X,σ), k a positive integer and $X=\bigcup_{i=1}^m A_i$. Let $f:X^k\longrightarrow X$ be a cyclic-Presic-Ciric operator. Then $\bigcap_{i=1}^m A_i\neq 0$ and f has a fixed point $x\in\bigcap_{i=1}^m A_i$, such that $\sigma(x,x)=0$. If in addition, the following conditions are satisfied:*

- *(i) $(f)\subset\cap_{i=1}^m Ai$;*
- *(ii) one of the following conditions is satisfied: (B_1) on the diagonal $\triangle\subset(\cap_{i=1}^m Ai)^k$;*

$$\sigma(f(x,...,x),f(y,...,y))<\sigma(x,y)$$

holds for all $x,y\in\cap_{i=1}^m Ai$ with $x\neq y$ or (B_2) in Definition (3.21)

Then the fixed point of f is unique.

3.12. Qawaqneh, Noorani, Shatanawi and Alsamir (2018). Qawaqneh et al. [41] established the existence of some common fixed point results for generalized Geraghty (α,ψ,ϕ)-quasi contraction self-mapping in partially ordered metric-like spaces.

Definition 3.22. [41] Let (X,σ) be a partially ordered metric-like space and $S,T:X\longrightarrow X$ be two mappings. Then we consider that the pair (f,g) is generalized Geraghty (α,ψ,ϕ)-quasi contraction self-mapping if there exist $\alpha:X\times X\to\mathbb{R}^+$, $\beta\in\mathcal{F},\psi\in\Psi$ and $\phi:\mathbb{R}^+\to\mathbb{R}^+$ are continuous functions with $\phi(t)\leq\psi(t)$ for all $t>0$ such that

$$\alpha(x,y)\psi(\sigma(fx,gy))\leq\lambda\beta(\psi(M(x,y))\phi(M(x,y)),$$

holds for all elements $x,y\in X$ and $0\leq\lambda<1$, where

$$M(x,y)=\max\{\sigma(x,y),\sigma(x,fx),\sigma(y,gy),\sigma(fx,y),\sigma(x,gy)\}$$

The main result of Qawaqneh et al [41] is the following.

Theorem 3.26. [41] *Let (X,σ) be a partially ordered metric-like space. Assume that $f,g:X\longrightarrow X$ are two self-mappings fulfilling the following conditions:*

- *(i) (f,g) is triangular α-admissible and there exists an $x_0\in X$ such that $\alpha(x_0,fx_0)\geq 1$;*
- *(ii) the pair (f,g) is weakly increasing;*
- *(iii) the pair (f,g) is a generalized Geraghty (α,ψ,ϕ)-quasi contraction non-self mapping;*
- *(iv) f and g are σ- continuous mappings.*

Then the pair (f, g) has a common fixed point $x \in X$ with $\sigma(x, x) = 0$. Moreover, assume that if $x_1, x_2 \in X$ such that $\sigma(x_1, x_1)$=$\sigma(x_2, x_2) = 0$ implies that x_1 and x_2 are comparable elements. Then the common fixed point of the pair (f, g) is unique.

Theorem 3.27. [41] *Let (X, σ) be a partially ordered metric-like space. Assume that $f, g : X \longrightarrow X$ are two self-mappings fulfilling the following conditions:*

- (i) *the (f, g) is triangular α-admissible;*
- (ii) *there exists an $x_0 \in X$ such that $\alpha(x_0, fx_0) \geq 1$;*
- (iii) *the pair (f, g) is a generalized Geraghty (α, ψ, ϕ)-quasi contraction non-self mapping;*
- (iv) *the pair (f, g) is weakly increasing;*
- (v) *if $\{x_n\}$ is a non-decreasing sequence in X such that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in \mathbb{N} \cup \{0\}$ and $x_n \longrightarrow u \in X$ as $n \to \infty$, then there exists a subsequence $\{x_{nl}\}$ of $\{x_n\}$ such that $x_n \preceq u$ for all l.*

Then the pair (f, g) has a common fixed point $x \in X$ with $\sigma(x, x) = 0$. Moreover, suppose that if $x_1, x_2 \in X$ such that $\sigma(x_1, x_1) = \sigma(x_2, x_x) = 0$ implies that x_1 and x_2 are comparable. Then, the common fixed point of the pair (f, g) is unique.

3.13. Qawaqneh, Noorani and Shantanawi (2018). Qawaqneh et al. [42] established the existence of some fixed point results for generalized (α, β, F)-Geraghty contraction in metric-like spaces.

Definition 3.23. [42] Let (X, σ) be a metric-like space and $\alpha : X \times X \to \mathbb{R}^+$. A mapping $T : X \longrightarrow X$ is said to be an (α, β, F)-Geraghty contraction mapping if there exist $\beta \in \mathcal{F}$ and $\tau > 0$ such that, for all $x, y \in X$ with $\sigma(Tx, Ty) > 0$ and $\alpha(x, y) \geq 1$,

$$\alpha(x, y)(\tau + F(\sigma(Tx, Ty))) \leq \beta(M(x, y))F(M(x, y)),$$

where,

$$M(x, y) = \max \left\{ \begin{array}{c} \sigma(x, y), \sigma(x, Tx), \sigma(y, Ty), \\ \frac{\sigma(Tx,y)+\sigma(x,Ty)}{4}, \frac{1+\sigma(x,Tx)\sigma(y,Ty)}{\sigma(x,y)+1} \end{array} \right\}.$$

The main result of Qawaqneh et al [42] is the following.

Theorem 3.28. [42] *Let (X, σ) be a metric-like space and $\alpha : X \times X \to \mathbb{R}^+$. A mapping $T : X \longrightarrow X$ be an α, β, F-Geraghty contraction mapping.Assume that the following conditions are satisfied:*

- (i) *$T \in \Xi(X, \alpha, \beta, F) \cap W\mathcal{A}(X, \alpha)$;*
- (ii) *there exists $x_0 \in X$ such that $\sigma(x_0, Tx_0) \leq 1$;*
- (iii) *T is σ-continuous .*

Then T has a unique fixed point $x \in X$ with $\sigma(x, x) = 0$

3.14. Alsamir, Noorani, Shantanawi, Aydi, Akhadkulov, Qawaqneh and Alanazi(2019). Alsamir et al. [5] established some fixed point results for (α, β)-admissible Z-contraction mappings in complete metric-like spaces.

Definition 3.24. [14] Let X be a nonempty set, $T : X \longrightarrow X$ and $\alpha, \beta : X \times X \to \mathbb{R}^+$. We say that T is an (α, β)-admissible mapping if $\alpha(x, y) \geq 1$ and $\beta(x, y) \geq 1$ imply that $\alpha(Tx, Ty) \geq 1$ and $\beta(Tx, Ty) \geq 1$ for all $x, y \in X$.

Definition 3.25. [29] A function $\zeta : \mathbb{R}^+ \times \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is called a simulation function if ζ satisfies the following conditions:

(ζ_1) $\zeta(0, 0)$ =0;
(ζ_2) $\zeta(t, s) < s - t$ for all $t, s > 0$;
(ζ_3) if $\{t_n\}$ and $\{s_n\}$ are sequences in $(0, \infty)$ such that $\lim_{n\to\infty} t(x_n)=\lim_{n\to\infty} s(x_n)=\ell \in \mathbb{R}^+$

then

$$\lim_{n\to\infty} \zeta(t_n, s_n) < 0.$$

Definition 3.26. [5] Let (X, σ) be a metric-like space. Given $T : X \longrightarrow X$ and $\alpha, \beta : X \times X \longrightarrow \mathbb{R}^+$ such that f is said to be an (α, β)-admissible Z-contraction with respect to ζ if

$$\zeta(\alpha(x, y)\beta(x, y)\sigma(Tx, Ty), \sigma(x, y)) \leq 0,$$

for all $x, y \in X$, where ζ is a simulation function.

The main result of Alsamir et al. [5] is the following.

Theorem 3.29. [5] *Let (X, σ) be a complete metric-like space and let T be a self-mapping on X satisfying the following Conditions:*

(i) T is (α, β)-admissible;
(ii) there exists $x_0 \in X$ such that $\sigma(x_0, fx_0) \geq 1$;
(iii) T is an (α, β)-admissible Z-contraction on (X, σ);
(iv) T is σ-continuous.

Then T has a unique fixed point $x \in X$ with $\sigma(x, x) = 0$.

Theorem 3.30. [5] *Let (X, σ) be a complete metric-like space and let T be a self-mapping on X satisfying the following Conditions:*

(i) T is (α, β)-admissible;
(ii) there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$ and $\beta(x_0, Tx_0) \geq 1$;
(iii) T is an (α, β)-admissible Z-contraction on (X, σ);
(iv) If $\{x_n\}$ is a sequence in X such that $\alpha(x_{nl}, x_{nl+1}) \geq 1$ and $\beta(x_{nl}, x_{nl+1}) \geq 1$, for all $l \in \mathbb{N}$ and $\alpha(x, Tx) \geq 1$ and $\beta(x, Tx) \geq 1$.

Then T has a unique fixed point $x \in X$ with $\sigma(x, x) = 0$.

3.15. Hammad and Sen (2019). Hammad and Sen [20] introduced the notion of α_L^ψ-rational contractive and cyclic α_L^ψ-rational contractive mapping and established the existence and uniqueness of fixed points for such mappings in complete metric-like spaces.

Definition 3.27. [20] Let (X, σ) be a metric-like space, $m \in \mathbb{N}$, $A_1, A_2, ..., A_m$ be σ-closed subsets of $X = \bigcup_{i=1}^m A_i$ and $\alpha : X \times X \longrightarrow \mathbb{R}^+$ be a mapping. We say that T is a cyclic α_L^ψ -rational contractive mapping if

(i) $T(A_j) \subseteq A_{j+1}, j = 1, 2, ..., m$, where $A_{q+1} = A_1$;

(ii) for any $x \in A_i$ and $y \in A_{i+1}, i = 1, 2, ..., m$, where $A_{m+1} = A_1$ and $\alpha(x, Tx)\alpha(y, Ty) \geq 1$, we get

$$\psi(\sigma(Tx, Ty)) \leq \psi(M_\sigma(x, y)) - LM_\sigma(x, y),$$

where $\psi \in \Psi$,$0 < L < 1$ and

$$M_\sigma(x,y) = \max\left\{ \begin{array}{c} \sigma(x,y), \frac{\sigma(x,Tx)\sigma(y,Ty)}{\sigma(x,y)}, \\ \frac{\sigma(y,Ty)(\sigma(x,Tx)+1)}{1+\sigma(x,y)}, \frac{\sigma(x,Ty)+\sigma(y,Tx)}{4} \end{array} \right\}.$$

If we take $X = A_i, i = 1, 2, ..., m$ in Definition 3.27, then we say T is an α_L^ψ-rational contractive mapping. We denote the set of all fixed points of T by $Fix(T)$, that is $Fix(T) = \{x \in X : Tx = x\}$.

Definition 3.28. Let (X, σ) be a metric-like space and $\alpha : X \times X \longrightarrow \mathbb{R}^+$. We say that an α-admissible mapping $T : X \longrightarrow X$ is α-continuous on (X, σ) if

$$x_n \to x \quad as \quad n \to \infty, \quad \alpha(x_n, x_{n+1}) \geq 1 \Rightarrow Tx_n \to Tx \quad for \quad all \quad n \in \mathbb{N}.$$

Theorem 3.31. [20] *Let (X, σ) be a complete metric-like space, m be a positive integer. $A_1, A2, ..., Am$ be nonempty σ-closed subsets of $X = \bigcup_{i=1}^m A_i$ and $\alpha : X \times X \longrightarrow \mathbb{R}^+$ be a mapping. Assume that $T : X \longrightarrow X$ is a cyclic α_L^ψ-rational contractive mapping satisfying the following conditions:*

(i) *T is an α-admissible mapping;*

(ii) *there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0) \geq 1$;*

(iii) *either T is α-continuous, or; for any sequence $\{x_n\}$ in X with $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \geq 0$ and $x_n \to x$ as $n \to \infty$, then $\alpha(x, Tx) \geq 1$. Then, S has a fixed point $x \in \bigcap_{i=1}^m A_i$. Moreover, if*

(iv) *for all $x \in Fix(T)$, we have $\alpha(x, x) \geq 1$. Then T has a unique fixed point $x \in \bigcap_{i=1}^m A_i$.*

3.16. Karapinar, Chen and Lee (2019). Karapinar et al. [28] established two best proximity point theorems in the setting of metric-like spaces that are based on cyclic contraction, Meir-Keeler-Kannan type cyclic contractions and generalized Ciric type cyclic ϕ-contraction via the M_T-function. Let $\mathcal{M}$ be the class of all functions $\gamma : \mathbb{R}^+ \to \mathbb{R}^+$ satisfying Definition 3.3.

By using the Kannan type cyclic contraction and Meir–Keeler function, we define the new notion of Meir–Keeler–Kannan type cyclic contraction, as follows:

Definition 3.29. [28] Let $\phi \in \mathcal{M}$ and $T : A \cup B \to A \cup B$ be a cyclic mapping, where A and B are nonempty subsets of a metric-like space (X, σ). Then, the mapping T is said to be a Meir-Keeler-Kannan type cyclic contraction, if

$$\sigma(Tx, Ty) - \sigma(A, B) \leq \phi\left(\frac{\sigma(x, Tx) + \sigma(y, Ty)}{2} - \sigma(A, B)\right)$$

for all $x \in A$ and $y \in B$.

Theorem 3.32. [28] *Let $T : A \cup B \to A \cup B$ be a cyclic-Meir-Keeler- Kannan type contraction, where A and B are nonempty closed subsets of a complete metric-like space (X, σ). If we construct a sequence $x_{n+1} = Tx_n$ for each $n \in \mathbb{N} \cup \{0\}$ for an arbitrary $x_0 \in A \cup B$, then we have the following:*

(i) *If $x_0 \in A$ and $\{x_{2n}\}$ has a subsequence $\{x_{2nk}\}$ which converges to $x^* \in A$ with $\sigma(x^*, x^*) = 0$, then $\sigma(x^*, Tx^*) = \sigma(A, B)$;*
(ii) *If $x_0 \in B$ and $\{x_{2n-1}\}$ has a subsequence $\{x_{2nk-1}\}$ which converges to $x^* \in B$ with $\sigma(x^*, x^*) = 0$, then $\sigma(x^*, Tx^*) = \sigma(A, B)$.*

Definition 3.30. [18] A mapping $T : A \cup B \to A \cup B$ is called cyclic if $T(A) \subset B$ and $T(B) \subset A$, where A, B are nonempty subsets of a metric space (X, d). In addition if there exists $k \in [0, 1)$ such that

$$d(Tx, Ty) \leq kd(x, y) + (1 - k)d(A, B),$$

for all $x \in A$ and $y \in B$, then the mapping T is called cyclic contraction.

Definition 3.31. A function $\psi : \mathbb{R}^+ \to [0, 1)$ is said to be an M_T-function, if

$$\lim_{s \to t^+} \sup \psi(s) = \inf_{a > 0} \sup_{0 < s - t < \alpha} \psi(s) < 1$$

for all $t \in \mathbb{R}^+$

Definition 3.32. [28] A mapping $T : A \cup B \to A \cup B$ is said to be a generalized M_T-Ciric-function type cyclic φ-contraction, if

$$\sigma(Tx, Ty) - \sigma(A, B) \leq \psi(\sigma(x, y))[\varphi(\sigma(x, y), \sigma(x, Tx), \sigma(y, Ty), \sigma(A, B)],$$

for all $x \in A$ and $y \in B$, where A and B are nonempty subsets of a metric-like space (X, σ) and ψ is an M_T-Function. Where M_T denotes set of all Meir-Keeler function.

Theorem 3.33. [28] *Let $T : A \cup B \to A \cup B$ be a generalized M_T-Ciric-function type cyclic φ-contraction, where A and B are nonempty closed subsets of a complete metric-like space (X, σ). If we construct a sequence $x_{n+1} = Tx_n$ for each $n \in \mathbb{N} \cup \{0\}$ for an arbitrary $x_0 \in A \cup B$, then we have the following:*

(i) *If* $x_0 \in A$ *and* $\{x_{2n}\}$ *has a subsequence* $\{x_{2nk}\}$ *which converges to* $x^* \in A$ *with* $\sigma(x^*, x^*) = 0$*, then* $\sigma(x^*, Tx^*) = \sigma(A, B)$*;*

(ii) *If* $x_0 \in B$ *and* $\{x_{2n-1}\}$ *has a subsequence* $\{x_{2nk-1}\}$ *which converges to* $x^* \in B$ *with* $\sigma(x^*, x^*) = 0$*, then* $\sigma(x^*, Tx^*) = \sigma(A, B)$*.*

3.17. Rao, Nashine and Kadelburg (2020). Rao et al. [44] discussed the existence of best proximity points of certain mappings via simulation functions in the frame of complete metric-like spaces.

Definition 3.33. [23] Let U and V be nonempty subsets of a metric-like spaces (X, σ) and $\alpha : U \times U \longrightarrow \mathbb{R}_0^+$ be a function. We say that the mapping T is α-proximal admissible if

$$\alpha(x, y) \geq 1 \quad \textit{and} \quad \sigma(u, Tx) = \sigma(v, Ty) = \sigma(U, V) \Rightarrow \alpha(u, v) \geq 1,$$

for all $x, y, u, v \in X$. If $\sigma(U, V) = 0$. Then T reduces from α-proximal admissible to α-admissible.

Definition 3.34. [27] Let $T : X \longrightarrow X$ be a mapping and $\alpha : X \times X \longrightarrow \mathbb{R}_0^+$ be a function. We say that the mapping T is triangular weakly-α-admissible if

$$\alpha(x, y) \geq 1 \quad \textit{and} \quad \alpha(y, z) \geq 1 \Rightarrow \alpha(x, z) \geq 1.$$

Definition 3.35. [44] Let (X, σ) be a metric-like space, U and V be two nonempty subsets of X, $\psi \in \Psi$, $\alpha : X \times X \longrightarrow \mathbb{R}_0^+$ and $\sigma \in Z$. We say that $T : U \longrightarrow V$ is an $\alpha - \psi - \sigma$-contraction if T is α-proximal admissible and

$$\alpha(x, y) \geq 1 \quad \textit{and} \quad \sigma(u, Tx) = \sigma(v, Ty) = \sigma(U, V)$$

$$\Rightarrow \varsigma(\alpha(x, y)\sigma(u, v), \psi(\sigma(x, y))) \geq 0,$$

for all $x, y, u, v \in U$.

Definition 3.36. [44] Let (X, σ) be a metric-like space, U and V be two nonempty subsets of X, $\alpha : X \times X \longrightarrow \mathbb{R}_0^+$ and $\sigma \in Z$. We say that $T : U \longrightarrow V$ is an $\alpha - \sigma$-contraction if T is α-proximal admissible and

$$\alpha(x, y) \geq 1 \quad \textit{and} \quad \sigma(u, Tx) = \sigma(v, Ty) = \sigma(U, V)$$

$$\Rightarrow \varsigma(\alpha(x, y)\sigma(u, v), \sigma(x, y))) \geq 0,$$

for all $x, y, u, v \in U$.

Theorem 3.34. [44] *Let* (X, σ) *be a metric-like space,* U *and* V *be two nonempty subsets of* X*,*$\alpha : X \times X \longrightarrow \mathbb{R}_0^+$*,* $\psi \in \Psi$ *and* $\sigma \in Z$*. is non-decreasing with respect to its second argument. Suppose that* $T : U \longrightarrow V$ *is an* $\alpha - \psi - \sigma$*-continuous and*

(i) T *is triangular weakly-*α*-admissible;*

(ii) *U is closed with respect to the topology T_δ;*
(iii) *$T(U_0) \subset V_0$;*
(iv) *there exist $x_0, x \in U$ such that $\sigma(x_1, Tx_0) = \sigma(U, V)$ and $\alpha(x_0, x_1) \geq 1$;*
(v) *T is continuous.*

Then T has a best proximity point, that is, there exists $z \in U$ such that $\sigma(z, Tz) = \sigma(U, V)$.

Definition 3.37. [44] Let (X, σ) be a metric-like space. U and V be two non-empty subsets of X,$\alpha : X \times X \longrightarrow \mathbb{R}_0^+$ and $\sigma \in Z$. We say that $T : U \longrightarrow V$ is a generalized $\alpha - \sigma$-contraction if T is α-proximal admissible,

$$\alpha(x, y) \geq 1 \quad and \quad \sigma(v, Ty) = \sigma(U, V)\sigma(\alpha(x, y)\sigma(u, v,)r(x, y) \geq 0,$$

for all $x, y, u, v \in U$ with $x \neq y$, where $r(x, y) = \max\left\{\sigma(x, y), \frac{\sigma(x,u)\sigma(y,v)}{\sigma(x,y)}\right\}$.

Theorem 3.35. [44] *Let (X, σ) be a metric-like space, U and V be two non-empty subsets of X,$\alpha : X \times X \longrightarrow \mathbb{R}_0^+$ and $\sigma \in Z$. Suppose $T : U \longrightarrow V$ is a generalized $\alpha - \sigma$-contraction and conditions $(i) - (v)$ of Theorem 3.34 are satisfied. Then T has a best proximity point.*

3.18. Mlaiki (2020). Mlaiki [34] introduced a new extension of the double controlled metric type spaces called double controlled metric-like spaces and generalized many results.

Definition 3.38. [24] Consider the set $X \neq \emptyset$ and a function $\mathfrak{h} : X \times X \rightarrow [1, \infty)$. Suppose that a function $\sigma : X \times X \longrightarrow \mathbb{R}^+$ satisfies the following conditions for all $x, y, z \in X$:

(i) $\sigma(x, y) = 0 \Leftrightarrow x = y$;
(ii) $\sigma(x, y) = \sigma(y, x)$;
(iii) $\sigma(x, y) \leq \mathfrak{h}(x, y)[\sigma(x, z) + \sigma(z, y)]$.

Then the pair (X, σ) is called an extended b-metric space.

Definition 3.39. [33] Given a nonempty set X and a function $\varpi : X \times X \rightarrow [1, \infty)$, suppose that a function $\rho : X \times X \rightarrow \mathbb{R}$ satisfies the following conditions for all $x, y, z \in X$:

(i) $\rho(x, y) = 0 \Leftrightarrow x = y$;
(ii) $\rho(x, y) = \rho(y, x)$;
(iii) $\rho(x, y) \leq \varpi(x, z)\rho(x, z) + \varpi(z, y)\rho(z, y)$.

Then the pair (X, ρ) is called a controlled metric-type space.

Definition 3.40. [2] Consider a set $X \neq \emptyset$ and non comparable functions $\varpi, \varepsilon : X \times X \rightarrow [1, \infty)$. Suppose that a function $\sigma : X \times X \rightarrow \mathbb{R}$ satisfies the following conditions for all $x, y, z \in X$:

(i) $\sigma(x,y)=0$ if and only if $x=y$;
(ii) $\sigma(x,y)=\sigma(y,x)$;
(iii) $\sigma(x,y)\leq\varpi(x,z)\sigma(x,z)+\varepsilon(z,y)\sigma(z,y)$.

Then the pair (X,σ) is called a double controlled metric type space.

Definition 3.41. [34] Consider a set $F\neq\emptyset$ and non comparable functions $\varpi,\varepsilon : F\times F\to[1,\infty)$. Suppose that a function $\sigma : F\times F\to\mathbb{R}^+$ satisfies the following conditions for all $g,h,\omega\in F$:

(i) $\sigma(x,y)=0\Rightarrow x=y$;
(ii) $\sigma(x,y)=\sigma(y,x)$;
(iii) $\sigma(x,y)\leq\varpi(x,z)\sigma(x,z)+\varepsilon(z,y)\sigma(z,y)$.

Then the pair (X,σ) is called a double controlled metric-like space.

Theorem 3.36. [34] *Let (X,σ) be a complete double controlled metric-like space defined by functions $\varpi,\varepsilon : X\times X\to[1,\infty)$. Let $T : X\to X$ be a mapping such that*

$$\sigma(Tx,Ty)\leq k\sigma(x,y)$$

for all $x,y\in X$, where $k\in(0,1)$. For $x_0\in X$, take $x_n=T^nx_0$. Suppose that

$$\sup_{m\geq1}\lim_{i\to\infty}\frac{\varpi(x_{i+1},x_{i+2})}{\varpi(x_i,x_{i+1})}\varepsilon(x_{i+1},x_m)<\frac{1}{k}.$$

Also, assume that for every $x\in X$, we have

$$\lim_{n\to\infty}\sigma(x,x_n)\quad and\quad\lim_{n\to\infty}\varepsilon(x_n,x)\quad exists\quad and\quad are\quad finite.$$

Then T has a unique fixed point.

Theorem 3.37. [34] *Let (X,σ) be a complete double controlled metric-like space defined by functions $\varpi,\varepsilon : X\times X\to[1,\infty)$. Consider a map $T : X\to X$ and assume that there exists a nondecreasing and continuous function $\phi:\mathbb{R}^+\longrightarrow\mathbb{R}^+$ such that*

$$\phi^i(x)\to0\quad as\quad n\to\infty, x>0, \sigma(Tx,Ty)\leq\phi(\Delta(x,y)),$$

$\Delta(x,y)=\max\{\sigma(x,y),\sigma(x,Tx),\sigma(y,Ty)\}$, for all $x,y\in X$. Moreover, assume that for each $x_0\in X$, we have

$$\sup_{m\geq1}\lim_{i\to\infty}\frac{\varpi(x_{i+1},x_{i+2})}{\varpi(x_i,x_{i+1})}\varepsilon(x_{i+1},x_m)\frac{\phi^{i+1}(\sigma(x_1,x_0))}{\phi^i(\sigma(x_i,x_0))}<1$$

where $x_n=T^nx_0$, $n\in\mathbb{N}$. If σ and T are continuous, then T has a unique fixed point.

3.19. Radenovic, Mirkov and Paunovic (2021). Radenovic et al. [43] generalized the two recently obtained results of Popescu, and Stan [40] regarding the F-contractions in complete, ordinary metric-like space to 0-complete partial metric-like space and 0-complete metric-like space. They also proved some new results in fixed point theory.

Definition 3.42. [55] Let (X, d) be a metric space. A mapping $T : X \longrightarrow X$ is called an F- contraction if there exists $\tau > 0$ such that

$$\tau + F(d(Tx, Ty)) \leq F(d(x, y)).$$

for all $x, y \in X$ with $d(Tx, Ty) > 0$,where F satisfies $F : (0, +\infty) \longrightarrow (-\infty, +\infty)$

- $(F_1 :)$ F is strictly increasing , i.e $0 < \alpha < \beta$ yields $F(\alpha) < F(\beta)$;
- $(F_2 :)$ for each sequence $\{\alpha_n\}_n \in \mathbb{N}$ in $(0, +\infty)$, $\lim_{n\to\infty} \alpha_n = 0$ if and only if $\lim_{n\to+\infty} F(\alpha_n) = -\infty$;
- (F_3) there exist $k \in (0, 1)$ such that $\lim_{\alpha\to 0^+} \alpha^k(\alpha) = 0$.

Definition 3.43. [16] Let (X, d) be a metric space. A mapping $T : X \longrightarrow X$ is called an F-contraction of Hardy-Rogers-type if there exist $\tau > 0$ and $F \in \mathcal{F}$ such that

$$\tau + F(d(Tx, Ty)) \leq F(A(x, y))$$

holds for any $x, y \in X$ with $d(Tx, Ty) > 0$ where $A(x, y) = \alpha.d(x, y) + \beta.d(x, Tx) + \gamma.d(y, Ty) + \delta.d(x, Ty) + L.d(y, Tx), \alpha, \beta, \gamma, \delta, L$ are non-negative numbers, $\gamma \neq 1$ and $\alpha + \beta + \gamma + 2\delta = 1$.

Theorem 3.38. [43] *Let T be a self-mapping of a 0-complete partial metric space (X, p). Suppose there exists $\tau > 0$ such that for all $x, y \in X$, $p(Tx, Ty) > 0$ yields*

$$\tau + F(p(Tx, Ty)) \leq F(A(x, y)),$$

where $F : (0, +\infty) \longrightarrow (-\infty, +\infty)$ is a strictly increasing mapping, $A(x, y) = \alpha.p(x, y) + \beta.p(x, Tx) + \gamma.p(y, Ty) + \delta.p(x, Ty) + L.p(y, Tx)$, $\alpha, \beta, \gamma, \delta, L$ are non-negative numbers $\delta < \frac{1}{2}$, $\gamma < 1$, $\alpha + \beta + \gamma + 2\delta + L = 1$, $0 < \alpha + \delta + L \leq 1$. Then T has a unique fixed point $x \in X$ and for every $x \in X$, the sequence $\{T^n x\}_{n\in\mathbb{N}\cup\{0\}}$ converges to x.

Theorem 3.39. [43] *Let T be a self-mapping of a 0-complete partial metric space (X, p). Suppose there exists $\tau > 0$ such that for all $x, y \in X$, $p(Tx, Ty) > 0$ yields*

$$\tau + F(p(Tx, Ty)) \leq F(p(x, y)),$$

where $F : (0, +\infty) \longrightarrow (-\infty, +\infty)$ is a mapping satisfying the conditions (*i*) *for each sequence* $\{\alpha_n\}_n \in \mathbb{N}$ *in* $\mathbb{R}^+$, $\lim_{n\to\infty} \alpha_n = 0$ *if and only if* $\lim_{n\to+\infty} F(\alpha_n) = -\infty$;

(*ii*) F *is continuous on* $(0, +\infty)$.

Then T *has a unique fixed point* $x \in X$ *and for every* $x \in X$ *the sequence* $\{T^n x\}_n \in \mathbb{N} \cup \{0\}$ *converges to* x.

Theorem 3.40. [43] *Let* T *be a self-mapping of* $0-\sigma$*-complete metric-like space* (X, σ). *Suppose that there exists* $\tau > 0$ *such that for all* $x, y \in X, \sigma(Tx, Ty) > 0$ *yields*

$$\tau + F(\sigma(Tx, Ty)) \leq F(A(x, y)),$$

where $F : (0, +\infty) \longrightarrow (-\infty, +\infty)$ *is a strictly increasing mapping,* $A(x, y) = \alpha.\sigma(x, y) + \beta.\sigma(x, Tx) + \gamma.\sigma(y, Ty) + \delta.\sigma(x, Ty) + L.\sigma(y, Tx), \alpha, \beta, \gamma, \delta, L$ *are non-negative numbers* $\delta < \frac{1}{2}$, $\gamma < 1$ $\alpha + \beta + \gamma + 2\delta + 2L = 1, 0 < \alpha + \beta + L \leq 1$. *Then* T *has a unique fixed point* $x \in X$, *and for every* $x \in X$, *the sequence* $\{T^n x\}_n \in N \cup \{0\}$ *converges to* x.

3.20. Mohammed, Alansari, Azam and Kanwal (2021). Mohammed et al. [39] introduced the notion of (φ, F)-Weak Contraction in the framework of metric-like spaces and established corresponding fixed point theorems.

Definition 3.44. [39] Let (X, σ) be a metric-like space, $F \in \mathcal{F}$ and $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ be a function satisfying the condition:

$$\lim_{p\to s^+} \inf \varphi(p) > 0 \quad for \quad all \quad s > 0.$$

A mapping $T : X \longrightarrow X$ is called a (φ, F)- weak contraction of type (A) if for all $x, y \in X$ for which $Tx \neq Ty$.

$$\varphi(\sigma(x, y)) + F(\sigma(Tx, Ty)) \leq F(\Omega_A(x, y))$$

where

$$\Omega_A(x, y) = \max\left\{\sigma(x, y), \sigma(x, Tx), \sigma(y, Ty), \frac{\sigma(x, Ty) + \sigma(y, Tx)}{2}\right\}.$$

The main theorem of Mohammed et al[39] is the following:

Theorem 3.41. [39] *Let* (X, σ) *be a 0-*σ*-complete metric-like space and* $T : X \longrightarrow X$ *be a* (φ, F)*-weak contraction of type* (A). *If* T *or* F *is continuous, then* T *has a unique fixed point in* X.

Definition 3.45. [39] Let (X, σ) be a metric-like space, $F \in \mathcal{F}$ and $\varphi : (0, \infty) \longrightarrow (0, \infty)$ be a function satisfying the condition:

$$\lim_{p\to s^+} \inf \varphi(p) > 0 \quad for \quad all \quad s > 0.$$

A mapping $T : X \longrightarrow X$ is called a (φ, F)- weak contraction of type (B) if for all $x, y \in X$ for which $Tx \neq Ty$.

$$\varphi(\sigma(x,y)) + F(\sigma(Tx,Ty)) \leq F(\Omega_B(x,y)).$$

where

$$\Omega_B(x,y) = \max\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty)\}.$$

Theorem 3.42. [39] *Let (X, σ) be a $0-\sigma$-complete metric-like space and $T : X \longrightarrow X$ be a (φ, F)-weak contraction of type (B). If T or F is continuous, then T has a unique fixed point in X.*

Definition 3.46. [39] Let (X, σ) be a metric-like space, $F \in \mathcal{F}$ and $\varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ be a function satisfying the condition:

$$\lim_{p \to s^+} \inf \varphi(p) > 0 \quad for \quad all \quad s > 0.$$

A mapping $T : X \longrightarrow X$ is called a (φ, F)- weak contraction of type (C) if for all $x, y \in X$ for which $Tx \neq Ty$.

$$\varphi(\sigma(x,y)) + F(\sigma(Tx,Ty)) \leq F(\Omega_C(x,y))$$

where

$$\Omega_C(x,y) = \max\left\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty), \frac{\sigma(x,Ty) + \sigma(y,Tx)}{2}\right\}.$$

Theorem 3.43. [39] *Let (X, σ) be a 0-σ-complete metric-like space and $T : X \longrightarrow X$ be a (φ, F)-weak contraction of type (C). If T or F is continuous, then T has a unique fixed point in X.*

3.21. Aysegul (2021). Aysegul [12] generalized the fixed point theorem given in Mlaiki [34] using the concept of double controlled metric-like spaces. The main theorem of Aysegul is the following: Let (X, d) be a complete double controlled metric-like space with $\theta, \mu : X \times X \to \mathbb{R}$ and T be a self mapping satisfying Reich condition. That is, T satisfies

$$d(Tx,Ty) \leq \alpha d(x,y) + \beta d(x,Tx) + \gamma d(y,Ty)$$

where $\alpha,\ \beta,\ \gamma \in (0,1)$ with $\alpha+\beta+\gamma < 1$. Let $r = \frac{\alpha+\beta}{1-\gamma} < 1$ for all $x, y \in X$. For all $x_0 \in X$, choose $x_n = T^n x_0$. Assume that

(*i*) $\sup_{m \geq 1} \lim_{i \to \infty} \frac{\theta(x_{i+1}, x_{i+2})}{\theta(x_i, x_{i+1})} .\mu(x_{i+1}, x_m) < \frac{1}{r}$;

(*ii*) $\lim_{n \to \infty} \theta(x, x_n) < \infty$ exists and finite and $\lim_{n \to \infty} \mu(x, x_n) < \dfrac{1}{\gamma}$.

Then T has a unique fixed point.

3.22. Shatanawi, Mlaiki, Rizk and Onunwor (2021).

Definition 3.47. [24] Consider the set $X \neq \emptyset$ and $\theta : X \times X \to [1, \infty)$. Let $d_e : X \times X \to \mathbb{R}$ be such that for all $x, y, z \in X$,

(*i*) $d_e(x, y) = 0$ if and only if $x = y$;
(*ii*) $d_e(x, y) = d_e(y, x)$;
(*iii*) $d_e(x, y) \leq \theta(x, y)[d_e(x, z) + d_e(z, x)]$.

Definition 3.48. [35] Consider the set $X \neq \emptyset$ and $\varrho : X \times X \to [1, \infty)$. Suppose that a function $d_c : X \times X \to [0, \infty)$ satisfies the following:

(CML_1) $d_c(x, y) = 0 \Rightarrow x = y$;
(CML_2) $d_c(x, y) = d_c(y, x)$;
(CML_3) $d_c(x, y) \leq \varrho(x, z)d(s, x) + \varrho(z, y)d_c(z, y)$ for all $x, y, z \in X$.

Then (X, d_c) is called a controlled metric-like space.

The main result of Shatanawi et al. [46] is the following:

Theorem 3.44. [46] *Let (X, d_c) be a complete controlled metric-like space, consider the mapping $T : X \times X$ such that*

$$d_c(Tx, Ty) \leq \vartheta(x)d_c(x, y),$$

for all $x, y \in X$, where $\vartheta \in A$. For $x_0 \in X$, take $x_n = T^n x_0$. Suppose that

$$\sup_{m \geq i} \lim_{i \to \infty} \frac{\varrho(x_{i+1}, x_{i+2})}{\varrho(x_i, x_{i+1})} \varrho(x_{i+1}, x_m) < \frac{1}{\vartheta(x_0)}.$$

Also, assume that for every $x \in X$, we have $\lim_{n \to \infty} \varrho(x_n, x)$ and $\lim_{n \to \infty} \varrho(x, x_n)$ exist and are finite. Then T has a unique fixed point.

Theorem 3.45. [46] *Let (X, d_c) be a complete controlled metric-like space by the function $\varrho : X \times X \longrightarrow [1, \infty)$. Let $T : X \longrightarrow X$, where*

$$d_c(Tx, Ty) \leq \vartheta(x)[d_c(x, Tx) + d_c(y, Ty)]$$

for all $x, y \in X$, where $\upsilon \in B$. For $x_0 \in X$, take $x_n = T^n x_0$. Suppose that

$$\sup_{m \geq 1} \lim_{i \to \infty} \frac{\varrho(x_{i+1}, x_{i+2})}{\varrho(x_{i+}, x_{i+1})} \varrho(x_{i+1}, x_m) < \frac{1 - \vartheta(x_0)}{\vartheta(x_0)}.$$

Also, assume that for every $x \in X$, we have $\lim_{n \to \infty} \varrho(x, x_n)$ exists, is finite and $\lim_{n \to \infty} \varrho(x_n, x) < \dfrac{1}{\vartheta(x_0)}$. Then there exists a unique fixed point of T.

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