

Mathematical analysis of regenerative point graphical technique (RPGT)

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ABSTRACT. In this paper, the behavioural analysis of a single-unit subdivision after a complete failure using the Regenerative Point Graphical Technique (RPGT) is discussed. Initially, the unit is fully operational which may have two types of malfunction, one is direct and the second is the partial failure mode. There is one server (fixer) that checks and fixes the unit for each failure. Incomplete failure of the unit cannot be restored to its original capacity. Each repair unit is subject to vandalism if the server reports that the unit cannot be repaired and replaced, following a normal distribution. The vague concept is used to determine unit failure/performance. Considering the levels of failure to be exponential, adjustment values are common and taking into account various possibilities, the system transformation diagram is developed to determine Primary, Second Circuits & Higher Education Foundations and Basic Status. The problem is constructed and resolved using RPGT to determine system parameters. System behaviour is discussed with the help of graphs.

1. Introduction

In the current business scenario, because of sizable improvement in technology, it turns into very stiff to preserve the working structures in properly operating condition. The complicated device and their availability for a long term turn out for any procedure industry. RPGT is one of the technique which is used to test the consistency of any operational device. It is the possibility that a factor will carry out its supposed characteristic safely for a designated duration beneath the required

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working situations. Availability is the possibility that an object while used beneath given situations will carry out satisfactorily while required. Maintainability is the possibility that an object may be restored to high-quality working situations inside a designated duration beneath said situations via way of means of employees having prescribed ability stage, sources and procedures. Then, it turns into very essential to screen renovation strategies. The overall performance of plant gadget may be better with reliability, availability and maintainability evaluation. If the working device is unavailable and unreliable then it lowers the performance of the plant. It ends in the failure of its manufacturing unit. In the sphere of engineering asset management, studies at the belongings fitness and existence span prediction has been multiplied.

Shakuntla et al. [9] discussed the reliability analysis of polytube using supplementary variable technique. The comparative analysis of the subsystem failed simultaneously was discussed by Shakuntla et al. [8]. Various Mechanical systems are assembly of a number of units and each unit is important for the system to work efficiently. If a single unit fails then the whole system fails. Considering the importance of individual units in system Kumar, J. & Malik, S.C.[1] have discussed the concept of preventive maintenance for a single unit system. Poonam and Pardeep Goel [6] discussed the sensitive analysis of biscuit plant. Rachna et al. [5] analyzed a three unit system with demand dependent operability of units. Shakuntla *et al.* [7] analyzed a non-markovian process by the inclusion of supplementary variable technique. Sunita et al. [2] analysed the behavior and profit analysis of thresher plant under study state. Sunita et al. [3] discussed RAP via constraint optimization genetic algorithm. Amardeep Singh and Pardeep Goel [11] discussed a case study of rico manufacturing plant using RPGT. Sunita et al. [4] find the solution of constrained problems using particle swarm optimization.

RAM (Reliability, Availability, Maintainability) of threshing machine in agriculture observed by Anchal et al. [10]. Degradation reduces the gadget's existence at distinct durations of time and because of its reliability of the device reduced. During reliability evaluation, it turns into very essential to research the degradation stage of belongings also. The theoretical idea can enhance the existence of belongings however want arises to apply a few mathematical evaluation.

Reliability evaluation is a totally fruitful approach which used a few mathematical approaches to discover the supply of the device. This approach facilitates in identifying the priorities of renovation to the gadget. The precedence can be given to the gadget which has a most failure charge. With positive regarded values of failure and restore charge, the optimization strategies are applied to calculate the high-quality mixture of failure and restore charge for the supply of the device. Later on, this facilitates in making plans renovation strategies/guidelines to hold the belongings in properly operating situations. So that those will make certain more

consistency of overall performance for a protracted tenure. Ultimately, the overall performance of a plant may be multiplied on this way.

2. Notations

pr/pf: Probability/transition probability factor of transition.

p_i, j : $p_i, j = q(i, j)^*(0)$; where * denotes Laplace transformation.

$\overline{p_i, j.k}$: $\overline{p_i, j.k} = q(i, j.k)^*(0)$; where * denotes Laplace transformation.

cycle represents a cycle through failure free states.

$(i \xrightarrow{s_k} j)$ is k-th directional path from state i to state j ; k takes a positive integer values for possible paths from state i to state j .

$(\xi \xrightarrow{sf} i)$ is a directional failure free path from base state ξ to state i .

$V_{(k,k)}$ represents transitional Probability Density Function of the state k reachable from the terminal state k of the k -cycle.

$V_{\overline{m,m}}$: Probability factor of reaching to the m^{th} terminal state from the $\overline{m-cycle}$.

$A_i(t)$ denotes the Availability of system in reduced or full state up time at time t .

$B_i(t)$ is fractional Busy Period of repairman or server for doing a particular job.

$V_i(t)$ is the fractional expected number of repairman's visits for doing a particular job in $(0, t]$ given that the system entered regenerative state i .

η_i is expected waiting time that repairman spends while doing a given job, given that the system entered regenerative state i at $t = 0$; $\eta_i = W_i^*(0)$.

μ_i : Mean sojourn time spent in state i , before visiting any other states

$$\int_0^\infty R_i(t) dt$$

and is determined by Laplace transformation of $R_i(t)$;

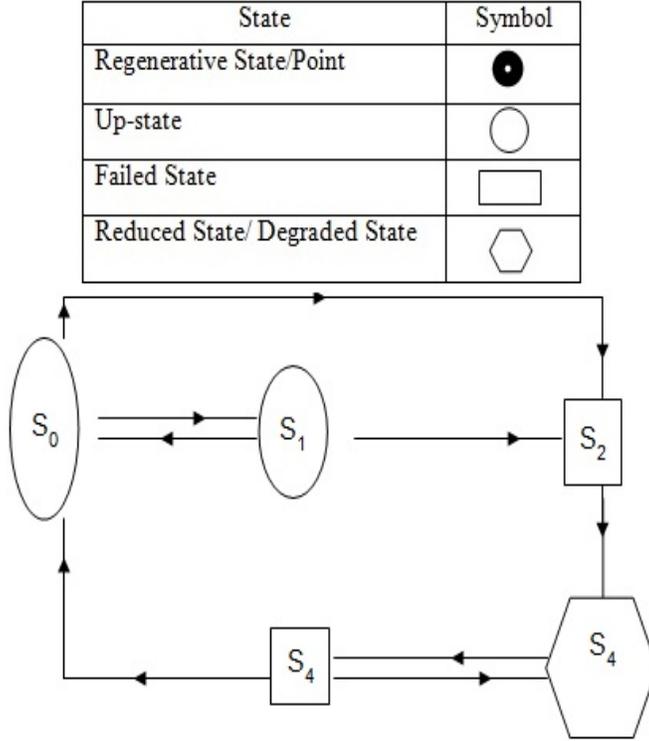
f_j is fuzziness value of the j -state. $f_j = 1$ for working state and $f_j = 0$ for failed state.

$q_{i,j}(t)$ stands for p.d.f. of system enter from a regenerative state i to any regenerative/failed state j without going through another state in $(0, t]$.

$p_{i,j}$ gives the steady state probability of system for transition from a regenerative state i to a regenerative state j without going through another regenerative state.

$p_{i,j}$ is determined by Laplace transformation of $q_{i,j}(t)$.

$R_i(t)$: Probability density function (p.d.f.) of the system that at time t , the system stay in regenerative state i .



3. Formulas used for calculations

Steady State Availability of the System:

It is defined as the proportion of time that the system is operational when the time-interval is very-very large and the corrective, preventive maintenance down times and the waiting times are included.

$$A_0 = \frac{MTBM}{(MTBM + MDT)}$$

where, MTBM = mean time between maintenance; MDT(mean down time) = statistical mean of the down times caused due to breakdowns, including supply down time, administrative down time.

The state transition diagram takes into account all the times under consideration of the stochastic system/process (for long run). Therefore, $\sum_j V_{0,j} \cdot \mu_j$ is the measure of the numerator and $\sum_i V_{0,i} \cdot \mu_i^1$ is the measure of denominator, where 'j' is a reachable un-failed and 'i' is a regenerative state in the state-transition diagram of the system. μ_i^1 is the total un-conditional time spent before transiting to any other regenerative state(s), given that the system entered regenerative state 'i' at $t = 0$.

Thus, steady state availability of a system is given by

$$A_0 = \frac{\left[\sum_j V_{0,j} \cdot \mu_j \right]}{\left[\sum_i V_{0,i} \cdot \mu_i^1 \right]}.$$

In case the system fails partially and is not fully available for its purpose then the availability of the system is discounted according to the proportions to the fuzziness measure of the states that the system can visit. Accordingly, the steady state availability of a system is modified to

$$A_0 = \frac{\left[\sum_j V_{0,j} \cdot f_j \cdot \mu_j \right]}{\left[\sum_i V_{0,i} \cdot \mu_i^1 \right]}$$

where f_j is the fuzziness measure of the un-failed state 'j'.

Availability of the System (Ao):

$$A_0 = \frac{\left[\sum_j V_{0,j} \cdot f_j \cdot \mu_j \right]}{\left[\sum_i V_{0,i} \cdot \mu_i^1 \right]}.$$

Busy Period of the Repairman:

Busy period of the server (under steady state conditions) doing a given job is defined by

$$B_0 = \frac{MTTR}{(MTBM + MDT)}.$$

Where, MTTR = mean time to repair; MTBM = mean time between maintenance; MDT(mean down time) = statistical mean of the down times caused due to breakdowns, including supply down time, administrative down time.(MDT is replaced by M or MTTR as per the real situation to which the stochastic process is subjected during its operation).

M = mean maintenance down time due to breakdowns and preventive maintenance actions.

Mean time to repair(MTTR) is defined by $\sum_j V_{0,j} \cdot \eta_j$ where 'j' is a regenerative state at which the server is busy doing repairs and η_j is the expected time spent in the state j, given that he system entered regenerative state j at $t = 0$. $MTBM + MDT = \sum_i V_{0,i} \cdot \mu_i^1$ where 'i' is a regenerative state and μ_i^1 is the un-conditional

time spent before transiting to any other regenerative state, given that the system entered regenerative state 'i' at $t = 0$.

The busy period of the server (under steady state conditions) doing a given job is given by

$$B_0 = \frac{\left[\sum_j V_{0,j} \cdot \eta_j \right]}{\left[\sum_i V_{0,i} \cdot \mu_i^1 \right]}$$

$$B_0 = \frac{\left[\sum_{j,s_r} \frac{(pr(0 \xrightarrow{s_r} j)) \cdot \eta_j}{\prod_{k_1 \neq 0} (1 - V_{k_1, k_1})} \right]}{\left[\sum_{i,s_r} \frac{(pr(0 \xrightarrow{s_r} i)) \cdot \mu_i^1}{\prod_{k_2 \neq 0} (1 - V_{k_2, k_2})} \right]}$$

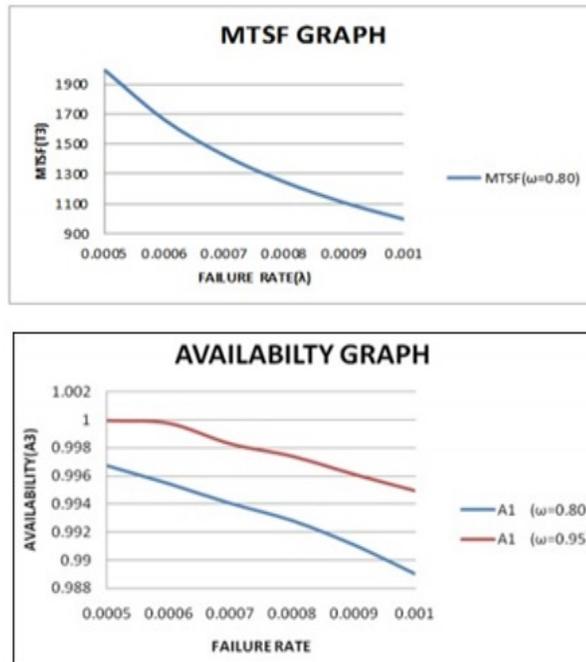
Number of Server's visits/number of Replacements: The expected number of visits of the server/replacements is defined by

$$V_0 = \frac{\left[\sum_j V_{0,j} \cdot \delta_j \right]}{\left[\sum_i V_{0,i} \cdot \mu_i^1 \right]}$$

where $\delta_j = 1$ if the visit of the server for the given job/replacement is a fresh at the regenerative state i , otherwise $\delta_j = 0$. The expected number of visits of the server/replacements is given by:

$$V_0 = \frac{\left[\sum_{j,s_r} \frac{(pr(0 \xrightarrow{s_r} j)) \cdot \eta_j}{\prod_{k_1 \neq 0} (1 - V_{k_1, k_1})} \right]}{\left[\sum_{i,s_r} \frac{(pr(0 \xrightarrow{s_r} i)) \cdot \mu_i^1}{\prod_{k_2 \neq 0} (1 - V_{k_2, k_2})} \right]}$$

4. Expected Graphs



5. Conclusion

From graph, if we increase in the repair rate the availability of the system and the mean time to system failure increases whereas if failure rate increases the availability and the mean time to system failure decreases. The RPGT is very useful for the evaluation of parameter in a simple way, without writing any state equation and without doing any lengthy and cumbersome calculations. It is easy for the management to control repair rate in comparison to failure rate. S

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