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A modified Taylor-series for solving a Fredholm integral equation of the second kind

Majid Amirfakhrian and Seyyed Mahmood Mirzaei*

ABSTRACT. In this paper, we propose a Taylor series expansion method for the second kind of Fredholm integral equation with smooth kernels. This method converts the integral equation into a linear equation system to reduce the amount of computation. We present convergence conditions. Finally, we show the efficiency of the method using some numerical examples.

1. Introduction

Considering the many applications that integral equations have in solving physical and mechanical problems, many efforts have been made to study solutions and the theory of integral equations [**3**, **14**]. Consider a second kind Fredholm Integral Equation:

$$x(s) = y(s) + \lambda \int_a^b k(s,t)x(t)dt, \tag{1}$$

where the parameter λ and the functions k and y are given, and x is the solution to be determined.

By many researchers and authors, numerical solutions have been studied for (1) and then presented [**1**, **4**, **5**, **9**, **10**, **23**]. Many of the integral equations in mathematics, physics, and engineering are solved using the Taylor series. Some authors have proposed solutions to equation (1) by the Taylor series [**11**, **12**, **13**, **15**, **16**, **17**, **18**, **19**, **20**, **21**, **22**] and it application [**6**], in this paper we propose a Taylor series expansion method for a second kind Fredholm integral equation.

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*Corresponding author

The existing numerical methods transform the integral equation into a linear system of algebraic equations that can be solved by direct or iterative methods. The linear system matrix can have a high order, which is difficult and time-consuming to find an exact or an approximate solution of the integral equation, and therefore methods have been developed more rapidly.

In this paper, a new modification of the Taylor series expansion method is proposed for the integral equations of the second kind (1), which has simple steps, is very effective and can provide an approximate solution for the integral equations shown in the examples. This method used to find the solution does not need to solve the system of linear equations, and more interestingly, this method does not even require all the Taylor series sentences. In the following sections, we will explain the specific and practical features of this method.

In Section 2, we present the main idea, some basic concepts, and Error bound of the work. Then, in Section 3, we will examine several specific cases. Using this method, a Fredholm integral equation of the second kind can be approximated. In Section 4, there are some examples.

2. Main Idea and its Application

In this section, we describe the new method for calculating the Fredholm integral equation of the second kind, numerically, which uses the development of the Taylor series.

Theorem 2.1. *For any even positive integer p, there is a $\eta \in (a,b)$, such that*

$$\int_a^b f(s)ds = \sum_{\substack{m=0 \\ m \text{ even}}}^{p} \frac{(b-a)^{m+1}}{2^m(m+1)!} f^{(m)}(\overline{s}) + R_{p+2}(f), \tag{2}$$

where $R_{p+2}(f) = \frac{(b-a)^{p+3}}{2^{p+2}(p+3)!} f^{(p+2)}(\eta)$ *and* $\overline{s} = \frac{b+a}{2}$.

Proof. [2]. □

Let in (1), for a positive even integer p we have $k \in C^p[a,b] \times C^p[a,b]$ and $y \in C^p[a,b]$. Thus $x \in C^p[a,b]$ [8].
Consider the Equation (1) as follows

$$x(s) - \lambda \int_a^b k(s,t)x(t)dt = y(s). \tag{3}$$

Let $q = \frac{p}{2}$. Using the numerical integration method (2) we have

$$x(s) - \lambda \sum_{m=0}^{q} \alpha_{2m} \frac{\partial^{2m}}{\partial t^{2m}}[k(s,\overline{t})x(\overline{t})] + E_{p+2} = y(s), \tag{4}$$

where

$$E_{p+2} = -\lambda \frac{(b-a)^{p+3}}{2^{p+2}(p+3)!} \frac{\partial^{p+2}}{\partial t^{p+2}} [k(s,t)x(t)]_{t=\eta}. \tag{5}$$

After omitting the error and by using the Leibnitz formula for derivation, we have

$$x(s) - \lambda \sum_{m=0}^{q} \sum_{j=0}^{2m} \alpha_{2m} \binom{2m}{j} \frac{\partial^{2m-j}}{\partial t^{2m-j}} k(s,\bar{t}) x^{(j)}(\bar{t}) = y(s). \tag{6}$$

Thus for $i = 0, 1, \ldots, p$, we have

$$x^{(i)}(s) - \lambda \sum_{m=0}^{q} \sum_{j=0}^{2m} \alpha_{2m} \binom{2m}{j} \frac{\partial^{i}}{\partial s^{i}} \frac{\partial^{2m-j}}{\partial t^{2m-j}} k(s,\bar{t}) x^{(j)}(\bar{t}) = y^{(i)}(s). \tag{7}$$

By changing the order of summations in the Equation (7) we have

$$x^{(i)}(s) - \lambda \sum_{j=0}^{p} \left(\sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \frac{\partial^{i}}{\partial s^{i}} \frac{\partial^{2m-j}}{\partial t^{2m-j}} k(s,\bar{t}) \right) x^{(j)}(\bar{t}) = y^{(i)}(s). \tag{8}$$

Let for $i, j = 0, 1, \ldots, p$;

$$c_{ij} = \lambda \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \frac{\partial^{i}}{\partial s^{i}} \frac{\partial^{2m-j}}{\partial t^{2m-j}} k(\bar{t},\bar{t}), \tag{9}$$

Thus

$$x^{(i)}(\bar{t}) - \sum_{j=0}^{p} c_{ij} x^{(j)}(\bar{t}) = y^{(i)}(\bar{t}). \tag{10}$$

Defining $A = (a_{ij})$, $b = (b_i)$ and $X = (\mathbf{x}_j)$, where

$$a_{ij} = \begin{cases} 1 - c_{ii}, & j = i; \\ -c_{ij}, & j \neq i. \end{cases} \qquad b_i = y^{(i)}(\bar{t}), \quad \mathbf{x}_j = x^{(j)}(\bar{t}), \tag{11}$$

one must solve the system of linear equations $AX = b$. In this case the approximated solution of the integral equation (3) from degree p is as follows:

$$x_p(s) = \sum_{j=0}^{p} \frac{\mathbf{x}_j}{j!} (s - \bar{t})^j. \tag{12}$$

Lemma 2.2. *Let for a positive even integer p we have $k \in C^p[a,b] \times C^p[a,b]$ and $y \in C^p[a,b]$. Also there exist two positive integers M and N such that for $s, t \in [a,b]$, and for $j = 0, 1, \ldots, p$, we have*

$$|x^{(j)}(s)| \leq M, \quad |\frac{\partial^j}{\partial t^j} k(s,t)| \leq N.$$

Then

$$|E_{p+2}| \leq |\lambda| M N \frac{(b-a)^{p+3}}{(p+3)!}.$$

Proof. For the error of this method using (5), we have

$$\begin{aligned} E_{p+2} &= -\lambda\frac{(b-a)^{p+3}}{2^{p+2}(p+3)!}\frac{\partial^{p+2}}{\partial t^{p+2}}[k(s,t)x(t)]_{t=\eta} \\ &= -\lambda\frac{(b-a)^{p+3}}{2^{p+2}(p+3)!}\sum_{j=0}^{p+2}\binom{p+2}{j}\frac{\partial^{p+2-j}}{\partial t^{p+2-j}}k(s,\eta)x^{(j)}(\eta). \end{aligned}$$

Thus

$$\begin{aligned} E_{p+2} \leq |\lambda|\frac{(b-a)^{p+3}}{2^{p+2}(p+3)!}MN\sum_{j=0}^{p+2}\binom{p+2}{j} &= |\lambda|\frac{(b-a)^{p+3}}{2^{p+2}(p+3)!}MN2^{p+2} \\ &= |\lambda|MN\frac{(b-a)^{p+3}}{(p+3)!}. \end{aligned}$$

□

The following corollary is an imidiate result of Lemma (2.2).

Corollary 2.3. *Let $k \in C^\infty[a,b]\times C^\infty[a,b]$ and $y \in C^\infty[a,b]$. Also there exist two positive integers M and N such that for $s,t \in [a,b]$, and for all $j = 0,1,\dots,$ we have*

$$|x^{(j)}(s)| \leq M, \quad |\frac{\partial^j}{\partial t^j}k(s,t)| \leq N.$$

Then x_p uniformly converges to x.

3. Special cases

In this section we consider some special cases. These special cases occur on $k(s,t)$, so that the derivative of order i from k with respect to s is a coefficient of itself and associated with i and the derivative of order j from k with respect to t is a coefficient of k and is related to j. Also, when the derivative of order i from k with respect to s and the derivative of order j from k with respect to t is equal to k, see below for more details.

- If $\frac{\partial^i}{\partial s^i}k(s,t) = \sigma^i k(s,t)$, then

$$c_{ij} = \lambda\sigma^i \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m}\binom{2m}{j}\frac{\partial^{2m-j}}{\partial t^{2m-j}}k(\bar{t},\bar{t}). \tag{13}$$

In this case we have

$$\det A = 1 - \sum_{i=1}^{p} c_{ii} = 1 - \lambda\sum_{i=1}^{p}\left(\sigma^i\sum_{m=[\frac{i+1}{2}]}^{q}\alpha_{2m}\binom{2m}{i}\frac{\partial^{2m-j}}{\partial t^{2m-j}}k(\bar{t},\bar{t})\right).$$

- If $\frac{\partial^j}{\partial t^j} k(s,t) = \gamma^j k(s,t)$, then

$$c_{ij} = \lambda \left(\sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \gamma^{2m-j} \right) \frac{\partial^i}{\partial s^i} k(\bar{t}, \bar{t}). \tag{14}$$

In this case we have

$$\det A = 1 - \sum_{j=1}^{p} c_{jj} = 1 - \lambda \sum_{j=1}^{p} \left(\sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \gamma^{2m-j} \right) \frac{\partial^j}{\partial s^j} k(\bar{t}, \bar{t}).$$

- If $\frac{\partial^i}{\partial s^i} k(s,t) = \sigma^i k(s,t)$ and $\frac{\partial^j}{\partial t^j} k(s,t) = \gamma^j k(s,t)$ then

$$c_{ij} = \lambda \sigma^i \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \gamma^{2m-j} k(\bar{t}, \bar{t}). \tag{15}$$

In this case we have

$$\det A = 1 - \sum_{i=1}^{p} c_{ii} = 1 - \lambda \sum_{i=1}^{p} \left(\sigma^i \sum_{m=[\frac{i+1}{2}]}^{q} \alpha_{2m} \binom{2m}{i} \gamma^{2m-i} \right) k(\bar{t}, \bar{t}).$$

Example 3.1. Let k be a kernel such that $\frac{\partial^i}{\partial s^i} k(s,t) = k(s,t)$ and $\frac{\partial^j}{\partial t^j} k(s,t) = k(s,t)$. Thus

$$c_{ij} = c_j = \lambda \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} k(\bar{t}, \bar{t}). \tag{16}$$

In this case we have

$$\det A = -\sum_{j=2}^{p} c_j = \lambda \left(\sum_{j=2}^{p} \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \right) k(\bar{t}, \bar{t}).$$

Thus if $k(\bar{t}, \bar{t}) \neq 0$ and $\lambda \neq 0$, thendet $A \neq 0$.
For example, if $k(s,t) = e^{s+t}$ and $a = -1, b = 1$, then

$$\det A = -\lambda \left(\sum_{j=2}^{p} \sum_{m=[\frac{j+1}{2}]}^{q} \alpha_{2m} \binom{2m}{j} \right).$$

Definition 3.2. A scale that used in numerical measurements is the root mean squared error (RMSE). To calculate the RMSE, the errors must be determined first. The difference between the exact values and the approximate values are the errors. They can be positive or negative because the approximate value can be less or more than the actual value. The higher the accuracy of the approximate values, the closer the RMSE will be to zero. In general, the RMSE value between the two approximate functions is lower, meaning that the approximate function is better. Suppose that we have n error values e computed as $(e_i; i = 1, 2, \cdots, n)$. For the given data, the RMSE is calculated as follow: [7]

$$RMSE = \sqrt{\frac{1}{r}\sum_{i=1}^{r} e_i^2} \quad , \tag{17}$$

in which r is the number of points applied to calculate RMSE.

4. Numerical Examples

In this section, we solve several integral equations by Modified Taylor series method. To do this, we used different values for p. We obtain the approximate solution for each of p and then we computed the RMSE at $r = 50$ points and present the results in a table.

Example 4.1. Consider the second kind linear Fredholm integral equation as follows:

$$x(s) = s + \frac{1}{2}\int_0^1 e^{\frac{s-t}{2}} x(t)dt,$$

with the exact solution $x(s) = s + (4\sqrt{e} - 6)e^{\frac{s-1}{2}}$.
We approximated the solution in 21 points and by using $p = 0, 4, 8, 10$ and $p = 12$. The errors in the equidistant points with step size $h = 0.1$ and for $p = 0, 4, 8, 10, 12$ are shown in the Table 1. Also RMSE results are shown in Table 2.

s	$p = 0$	$p = 4$	$p = 8$	$p = 10$	$p = 12$
0	0.639184	0.000159	4.06503×10^{-10}	2.88047×10^{-13}	0
0.1	0.520685	0.000165	4.22990×10^{-10}	3.00149×10^{-13}	5.55112×10^{-17}
0.2	0.401237	0.000172	4.44052×10^{-10}	3.15359×10^{-13}	5.55112×10^{-17}
0.3	0.280792	0.000180	4.66770×10^{-10}	3.31513×10^{-13}	0
0.4	0.159298	0.000190	4.90700×10^{-10}	3.48443×10^{-13}	0
0.5	0.036703	0.000199	5.15859×10^{-10}	3.66374×10^{-13}	5.55112×10^{-17}
0.6	0.087051	0.000210	5.42300×10^{-10}	3.85136×10^{-13}	5.55112×10^{-17}
0.7	0.212022	0.000220	5.70111×10^{-10}	4.04898×10^{-13}	1.11022×10^{-16}
0.8	0.338274	0.000231	5.99293×10^{-10}	4.25660×10^{-13}	1.11022×10^{-16}
0.9	0.465872	0.000242	6.29404×10^{-10}	4.47198×10^{-13}	0
1	0.594885	0.000252	6.57382×10^{-10}	4.68070×10^{-13}	8.88178×10^{-16}

Table 1. $\mid P(s_i) - x(s_i) \mid$ for Example 4.1 with different p

$p = 0$	$p = 4$	$p = 8$	$p = 10$	$p = 12$
0.364580	0.000204	5.26924×10^{-10}	3.74244×10^{-13}	1.39553×10^{-16}

TABLE 2. RMSE for Example 4.1 with $r = 50$

Example 4.2. Let $k(s,t) = \cos(s + 2t)$, and we Consider the second kind linear Fredholm integral equation as follow:

$$x(s) = \frac{2\Big(-2e\sin(s) + 2\sin(s+2) + e\cos(s) - \cos(s+2)\Big)}{5e} + 2\int_0^1 \cos(s+2t)x(t)dt,$$

with the exact solution $x(s) = e^{-s}$.
We approximated the solution in 21 points and by using $p = 0, 4, 6, 8$ and $p = 10$. The errors in the equidistant points with step size $h = 0.1$ and for $p = 0, 4, 6, 8$ and $p = 10$ are shown in the Table 3. Also RMSE results are shown in Table 4.

s	$p = 0$	$p = 4$	$p = 6$	$p = 8$	$p = 10$
0	0.598130	0.090446	0.001262	2.62197×10^{-6}	3.35422×10^{-7}
0.1	0.502967	0.085056	0.001160	2.88758×10^{-6}	3.15985×10^{-7}
0.2	0.416861	0.078717	0.001046	3.12691×10^{-6}	2.93358×10^{-7}
0.3	0.338948	0.071549	0.000923	3.33537×10^{-6}	2.67782×10^{-7}
0.4	0.268450	0.063652	0.000790	3.51054×10^{-6}	2.39517×10^{-7}
0.5	0.204660	0.055118	0.000649	3.65064×10^{-6}	2.08854×10^{-7}
0.6	0.146941	0.046032	0.000502	3.75426×10^{-6}	1.76103×10^{-7}
0.7	0.094715	0.036486	0.000350	3.82037×10^{-6}	1.41599×10^{-7}
0.8	0.047459	0.026578	0.000194	3.84828×10^{-6}	1.05693×10^{-7}
0.9	0.004699	0.016422	0.000036	3.83738×10^{-6}	6.87495×10^{-8}
1	0.033991	0.006149	0.000123	3.78584×10^{-6}	3.11502×10^{-8}

TABLE 3. $\mid P(s_i) - x(s_i) \mid$ for Example 4.2 with different p

$p = 0$	$p = 4$	$p = 6$	$p = 8$	$p = 10$
5.64609	0.574567	0.000747	3.51458×10^{-6}	2.19830×10^{-7}

TABLE 4. RMSE for Example 4.2 with $r = 50$

Example 4.3. Let $k(s,t) = s^2 + t$, and we consider the second kind linear Fredholm integral equation with the exact solution $x(s) = s^3 e^{-s} \ln(s + 2) \sin s$ on $[-1, 1]$ and $\lambda = 1$. We approximated the solution in 21 points and by using $p = 0, 8, 10, 12$ and $p = 24$. The errors in the equidistant points with step size $h = 0.2$

and for $p = 0, 8, 10, 12, 24$ are shown in the Table 5. Also RMSE results are shown in Table 6.

s	$p = 0$	$p = 8$	$p = 10$	$p = 12$	$p = 24$
-1	0.024212	0.003388	0.002289	0.000620	1.05480×10^{-7}
-0.8	0.173243	0.018877	0.001375	0.000363	2.74983×10^{-8}
-0.6	0.098986	0.015172	0.001147	0.000282	1.88473×10^{-8}
-0.4	0.041687	0.009833	0.000765	0.000185	1.23377×10^{-8}
-0.2	0.025353	0.006452	0.000529	0.000126	8.43176×10^{-9}
0	0.024212	0.005323	0.000450	0.000107	7.12978×10^{-9}
0.2	0.025283	0.006452	0.000529	0.000126	8.43176×10^{-9}
0.4	0.038838	0.009847	0.000765	0.000185	1.23377×10^{-8}
0.6	0.088169	0.015731	0.001165	0.000283	1.88478×10^{-8}
0.8	0.194132	0.026537	0.001841	0.000445	2.81684×10^{-8}
1	0.364298	0.056326	0.003827	0.001032	9.09111×10^{-8}

Table 5. $\mid P(s_i) - x(s_i) \mid$ for Example 4.3 with different p

$p = 0$	$p = 8$	$p = 10$	$p = 12$	$p = 24$
0.126576	0.018104	0.001313	0.000333	2.76302×10^{-8}

Table 6. RMSE for Example 4.3 with $r = 50$

Example 4.4. In this example we use [18] and solve one of its examples with our method. As you can see below, although we did not increase the value of p much, the results of our method are better than the results of that method.
Now let $k(s,t) = s + t$, and we consider the second kind linear Fredholm integral equation with the exact solution $x(s) = e^s$ on $[0, 1]$ and $\lambda = 1$. We approximated the solution in 21 points and by using $p = 4, 8$. The errors in the equidistant points with step size $h = 0.1$ and for $p = 4, 8$ are shown in the Table 7. Also RMSE results are shown in Table 8.

5. Conclusion

In this study, we introduced a modified Taylor series expansion method developed an efficient and computationally attractive method to solve the Fredholm integral equations of the second kind. In this work, the Taylor series expansion method has been successfully applied to find the solution of Fredholm integral equations of the second kind. An error analysis for the method is also provided. This method can

s	$p=4$	$p=8$
0	0.00062	1.11572×10^{-8}
0.1	0.00040	4.38613×10^{-9}
0.2	0.00034	3.86162×10^{-9}
0.3	0.00035	4.31023×10^{-9}
0.4	0.00039	4.84139×10^{-9}
0.5	0.00044	5.37483×10^{-9}
0.6	0.00048	5.90827×10^{-9}
0.7	0.00052	6.43935×10^{-9}
0.8	0.00053	6.88298×10^{-9}
0.9	0.00046	6.26814×10^{-9}
1	0.00018	1029658×10^{-9}

Table 7. $\mid P(s_i)-x(s_i)\mid$ for Example 4.4

$p=4$	$p=8$
0.00044	5.60866×10^{-9}

Table 8. RMSE for Example 4.4 with $r=50$

be concluded that the method is useful and efficient techniques in finding solutions for wide classes of problems.

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Department of Mathematics, Central Tehran Branch, Islamic Azad University, Tehran, Iran

Email address: amirfakhrian@iauctb.ac.ir

Department of Mathematics, Gonbad Kavoos Branch, Islamic Azad University, Gonbad Kavoos, Iran

Email address: s.mahmoodmirzaei@gmail.com

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