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Intuitionistic fuzzy stability of the heptic functional equation

Mohammad Shafii Mousavi

Abstract. In this article, by using a fixed point method, we establish the intuitionistic fuzzy version of Hyers-Ulam stability for a heptic functional equation that was introduced by Xu and Rassias. This way shows that the concept of stability is related to some fixed point of a suitable operator.

1. Introduction

In 1965, Zadeh [32] introduced the notion of fuzzy sets was which is a powerful hand set for modeling uncertainty and vagueness in various problems arising in the field of science and engineering. After that, fuzzy theory has become very active area of research and a lot of developments have been made in the theory of fuzzy sets to find the fuzzy analogues of the classical set theory. In fact, a large number of research papers have appeared by using the concept of fuzzy set and numbers and also fuzzification of many classical theories has been made. The concept of intuitionistic fuzzy normed spaces, initially has been introduced by Saadati and Park in [24]. Then, Saadati et al. have obtained a modified case of intuitionistic fuzzy normed spaces by improving the separation condition and strengthening some conditions in the definition of [26]. Many authors have considered the intuitionistic fuzzy normed linear spaces, and intuitionistic fuzzy 2-normed spaces (see [2], [3], [5], [15], [13], [19]).

In [27], Ulam proposed the general Ulam stability problem: "When is it true that by slightly changing the hypotheses of a theorem one can still assert that the thesis of the theorem remains true or approximately true?" In [14], Hyers gave the first affirmative answer to the question of Ulam for additive functional equations on Banach spaces. On the other hand, Cădariu and Radu noticed that a fixed point alternative method is very important for the solution of the Ulam problem. In other

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words, they employed this fixed point method to the investigation of the Cauchy functional equation [11] and for the quadratic functional equation [10] (for more applications of this method, see [6], [7], [9] and [31]). The generalized Hyers-Ulam stability of different functional equations in intuitionistic fuzzy normed spaces has been studied by a number of the authors (see [1], [4], [8], [17], [18], [22], [23] and [29])

Xu and Rassias [28] introduced the following heptic functional equation:

$$f(x+4y)-7f(x+3y)+21f(x+2y)-35f(x+y)+35f(x)-21f(x-y) \\ +7f(x-2y)-f(x-3y)=5040f(y). \quad (1)$$

It is easy to check that the function $f(x)=ax^7$ is a solution of the functional equation (1). In this paper, we investigate some stability results concerning the functional equation (1) in the setting of intuitionistic fuzzy normed spaces by a fixed point alternative method; for an application of this method see [16].

2. Preliminaries results

In this section, we restate the usual terminology, notations and conventions of the theory of intuitionistic fuzzy normed space, as in [20], [21], [22], [23] and [25].

Let $\leq_L$ be an order relation on the set $L=\{(x_1,x_2):(x_1,x_2)\in[0,1]^2, x_1+x_2\leq 1\}$ defined by

$$(x_1,x_2)\leq_L(y_1,y_2)\Longleftrightarrow x_1\leq y_1, y_2\leq x_2$$

for all $(x_1,x_2),(y_1,y_2)\in L$. It is easy to check that the pair $(L,\leq_L)$ is a complete lattice (see also [21] and [25]). We denote the units of L by $0_L=(0,1)$ and $1_L=(1,0)$.

Definition 2.1. Let U be a non-empty set called the universe. An L-fuzzy set in U is defined as a mapping $\mathcal{F}:U\longrightarrow L$. For each u in U, $\mathcal{F}(u)$ represents the degree (in L) to which u is an element of $\mathcal{F}$. An intuitionistic fuzzy set $\mathcal{F}_{\mu,\nu}$ in a universal set U is an object $\mathcal{F}_{\mu,\nu}=\{(\mu_{\mathcal{F}}(u),\nu_{\mathcal{F}}(u)):u\in U\}$, where $\mu_{\mathcal{F}}(u)$ and $\nu_{\mathcal{F}}(u)$ belong to $[0,1]$ for all $u\in U$ with $\mu_{\mathcal{F}}(u)+\nu_{\mathcal{F}}(u)\leq 1$. The numbers $\mu_{\mathcal{F}}(u)$ and $\nu_{\mathcal{F}}(u)$ are called the membership degree and the non-membership degree, respectively, of u in $\mathcal{F}_{\mu,\nu}$.

Definition 2.2. A triangular norm (t-norm) on L is a mapping $\mathcal{T}:L\times L\longrightarrow L$ satisfying the following conditions:

(i) $\mathcal{T}(x,1_L)=x$ (boundary condition) $\quad(x\in L)$;
(ii) $\mathcal{T}(x,y)=\mathcal{T}(y,x)$ (commutativity) $\quad(x,y\in L)$;
(iii) $\mathcal{T}(x,\mathcal{T}(y,z))=\mathcal{T}(\mathcal{T}(x,y),z)$ (associativity) $\quad(x,y,z\in L)$;
(iv) $x_1\leq_L y_1$ and $x_2\leq_L y_2\Longrightarrow\mathcal{T}(x_1,x_2)\leq_L\mathcal{T}(y_1,y_2)$ (monotonicity) $\quad(x_1,x_2,y_1,y_2\in L)$.

A t-norm $\mathcal{T}$ on L is said to be *continuous* if, for any $x, y \in L$ and any sequences $\{x_n\}$ and $\{y_n\}$ which converge to x and y, respectively, $\lim_{n\to\infty} \mathcal{T}(x_n, y_n) = \mathcal{T}(x, y)$. For $x = (x_1, x_2), y = (y_1, y_2) \in L$, $\mathcal{T}(x, y) = (x_1y_1, \min\{x_2 + y_2, 1\})$ and $\mathcal{M}(x, y) = (\min\{x_1, y_1\}, \max\{x_2, y_2\})$ are continuous t-norm [29].

Here, we define a sequence $\mathcal{T}^n$, recursively by $\mathcal{T}^1 = \mathcal{T}$ and

$$\mathcal{T}^n\left(x^{(1)}, x^{(1)}, \cdots, x^{(n+1)}\right) = \mathcal{T}\left(\mathcal{T}^{n-1}\left(x^{(1)}, x^{(1)}, \cdots, x^{(n)}\right), x^{(n+1)}\right)$$

for all $n \geq 2$ and $x^{(j)} \in L$.

Definition 2.3. A negator on L is a decreasing mapping $\mathfrak{N} : L \longrightarrow L$ satisfying $\mathfrak{N}(0_L) = 1_L$ and $\mathfrak{N}(1_L) = 0_L$. If $\mathfrak{N}(\mathfrak{N}(x)) = x$, for all $x \in L$, then $\mathfrak{N}$ is called an involutive negator. A negator on $[0, 1]$ is a decreasing mapping $\mathcal{N} : L \longrightarrow L$ satisfying $\mathcal{N}(0) = 1$ and $\mathcal{N}(1) = 0$. The standard negator on $[0, 1]$ is defined by $\mathcal{N}_s(x) = 1 - x$ for all $x \in [0, 1]$.

The following defnitions of an intuitionistic fuzzy normed space are taken from [22].

Definition 2.4. Let $\mathcal{L} = (L, \leq_L)$. Let X be a vector space, $\mathcal{T}$ be a continuous t-norm on L and $\mathcal{P}$ be an L-fuzzy set on $X \times (0, \infty)$ satisfying the following conditions:

(i) $0 <_L \mathcal{P}(x, t)$;
(ii) $\mathcal{P}(x, t) = 1_L$ if and only if $x = 0$;
(iii) $\mathcal{P}(\alpha x, t) = \mathcal{P}\left(x, \frac{t}{|\alpha|}\right)$ for all $\alpha \neq 0$;
(iv) $\mathcal{T}(\mathcal{P}(x, t), \mathcal{P}(y, s)) \leq_L \mathcal{P}(x + y, t + s)$;
(v) The map $\mathcal{P}(x, \cdot) : (0, \infty) \longrightarrow L$ is continuous;
(vi) $\lim_{t\to 0} \mathcal{P}(x, t) = 0_L$ and $\lim_{t\to\infty} \mathcal{P}(x, t) = 1_L$;

for all $x, y \in X$ and all $t, s > 0$. Then the triple $(X, \mathcal{P}, \mathcal{T})$ is called an L-fuzzy normed space. In this case $\mathcal{P}$ is called $\mathcal{L}$-fuzzy norm (briefly, L-fuzzy norm). If $\mathcal{P} = \mathcal{P}_{\mu,\nu}$ is an intuitionistic fuzzy set, then the triple $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is said to be an intuitionistic fuzzy normed space (briefly, IFN-space). In this case, $\mathcal{P}_{\mu,\nu}$ is called an intuitionistic fuzzy norm on X (Some examples of IFN-spaces are provided in [29] and [31]).

Note that, if $\mathcal{P}$ is an L-fuzzy norm on X, then the following statements hold:

(i) $\mathcal{P}(x, t)$ is nondecreasing with respect to t for all $x \in X$;
(ii) $\mathcal{P}(x - y, t) = \mathcal{P}(y - x, t)$ for all $x, y \in X$ and $t > 0$.

Example 2.5. [29] Let $(X, \|\cdot\|)$ be a normed space. Let $\mathcal{T}(x, y) = (x_1y_1, \min\{x_2+ y_2, 1\})$ for all $x = (x_1, x_2), y = (y_1, y_2) \in L$ and μ, ν be membership and non-membership degree, respectively, of an intuitionistic fuzzy set defined by

$$\mathcal{P}_{\mu,\nu}(x, t) = (\mu(x, t), \nu(x, t)) = \left(\frac{t}{t + \|x\|}, \frac{\|x\|}{t + \|x\|}\right) \qquad (t \in \mathbb{R}^+).$$

Then $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is an IFN-space.

Definition 2.6. Let $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ be an IFN-space.

(1) A sequence $\{x_n\}$ in $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is said to be *convergent* to a point x if $\mathcal{P}_{\mu,\nu}(x_n - x, t) \to 1_L$ as $n \to \infty$ for all $t > 0$;
(2) A sequence $\{x_n\}$ in $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is called a *Cauchy sequence* if, for every $t > 0$ and $0 < \epsilon < 1$, there exists a positive integer N such that $(N_s(\epsilon), \epsilon) \leq_L \mathcal{P}_{\mu,\nu}(x_n - x_m, t)$ for all $m, n > N$, where N_s is the standard negator.;
(3) $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is said to be *complete* if and only if every Cauchy sequence in $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is convergent to a point in $(X, \mathcal{P}_{\mu,\nu}, \mathcal{T})$. A complete intuitionistic fuzzy normed space is called an intuitionistic fuzzy Banach space.

We bring the following theorem which is a fundamental result in fixed point theory [12]. This result plays a fundamental role to arrive our purpose in this paper.

Theorem 2.1. (*The fixed point alternative theorem*) *Let (Δ, d) be a complete generalized metric space and $\mathcal{J} : \Delta \longrightarrow \Delta$ be a mapping with Lipschitz constant $L < 1$. Then, for each element $\alpha \in \Delta$, either $d(\mathcal{J}^n\alpha, \mathcal{J}^{n+1}\alpha) = \infty$ for all $n \geq 0$, or there exists a natural number n_0 such that*

(i) *$d(\mathcal{J}^n\alpha, \mathcal{J}^{n+1}\alpha) < \infty$ for all $n \geq n_0$;*
(ii) *the sequence $\{\mathcal{J}^n\alpha\}$ is convergent to a fixed point β^* of $\mathcal{J}$;*
(iii) *β^* is the unique fixed point of $\mathcal{J}$ in the set $\Delta_1 = \{\beta \in \Delta : d(\mathcal{J}^{n_0}\alpha, \beta) < \infty\}$;*
(iv) *$d(\beta, \beta^*) \leq \frac{1}{1-L} d(\beta, \mathcal{J}\beta)$ for all $\beta \in \Delta_1$.*

3. Intuitionistic fuzzy stability of (1)

In this section, we prove the Ulam-Hyers stability of the equation (1) in intuitionistic fuzzy normed spaces, based on Theorem 2.1. From now on, we use the difference operator for the given mapping $f : X \longrightarrow Y$ as follows:

$$\begin{aligned} D_h f(x, y) &= f(x + 4y) - 7f(x + 3y) + 21f(x + 2y) - 35f(x + y) + 35f(x) \\ &\quad - 21f(x - y) + 7f(x - 2y) - f(x - 3y) - 5040f(y) \end{aligned}$$

for all $x, y \in X$.

The following theorem which is proved in [30, Theorem 2.1]. Note that some basic facts on n-additive symmetric mappings can be found in [28].

Theorem 3.1. *Let X and Y be vector spaces. A function $f : X \longrightarrow Y$ is a solution of the functional equation (1) if and only if f is of the form $f(x) = A^7(x)$ for all $x \in X$, where $A^7(x)$ is the diagonal of the 7-additive symmetric map $A_7 : X^7 \longrightarrow Y$.*

Here and subsequently, we assume that all t-norms are as $\mathcal{T}(x,y) = (\min\{x_1,y_1\}, \max\{x_2,y_2\})$ for all $x=(x_1,x_2), y=(y_1,y_2)\in L$. In the next theorem, we prove the Ulam-Hyers stability of the equation (1) in intuitionistic fuzzy normed spaces.

Theorem 3.2. *Let $l\in\{1,-1\}$ be fixed and let α be a real number with $\alpha\neq 128$. Let X be a linear space and let $(Z,\mathcal{P}'_{\mu,\nu},\mathcal{T}')$ be an intuitionistic fuzzy normed space. Suppose that $\phi : X\times X\longrightarrow Z$ is a mapping such that*

$$\mathcal{P}'_{\mu,\nu}(\alpha^l\phi(x,y),t)\leq_L \mathcal{P}'_{\mu,\nu}(\phi(2^l x,2^l y),t) \tag{2}$$

for all $x\in X$ and $t>0$. If $(Y,\mathcal{P}_{\mu,\nu},\mathcal{T})$ is a complete intuitionistic fuzzy normed space and $f : X\longrightarrow Y$ is a mapping such that

$$\mathcal{P}'_{\mu,\nu}(\phi(x,y),t)\leq_L \mathcal{P}_{\mu,\nu}(D_h f(x,y),t) \tag{3}$$

for all $x,y\in X$ and $t>0$, then there exists a unique heptic mapping $H : X\longrightarrow Y$ such that

$$\Lambda(x,|128-\alpha|t)\leq_L \mathcal{P}_{\mu,\nu}(H(x)-f(x),t) \tag{4}$$

for all $x\in X$ and $t>0$, where

$$\begin{aligned}\Lambda(x,t) :=\ &\mathcal{T}^{30}(\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\tfrac{63}{4}t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\tfrac{63}{4}t),\mathcal{P}'_{\mu,\nu}(\phi(0,6x),79380t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),79380t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),11340t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),11340t)\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,2x),3780t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),3780t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),4536t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\tfrac{45}{4}t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),56700t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\tfrac{132300}{11}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\tfrac{132300}{11}t),\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\tfrac{105}{22}t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\tfrac{37800}{11}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,x),2700t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),2700t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),18900t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),18900t),\mathcal{P}'_{\mu,\nu}(\phi(x,x),\tfrac{1260}{167}t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),1800t)\\
&\mathcal{P}'_{\mu,\nu}(\phi(-x,x),\tfrac{1260}{167}t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),18900t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),18900t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,3x),9450t),\mathcal{P}'_{\mu,\nu}(\phi(3x,-3x),9450t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),900t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),900t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),540t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),540t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,0),1080t)).\end{aligned} \tag{5}$$

Proof. For the cases $l=1$ and $l=-1$, we consider $\alpha<128$ and $\alpha>128$, respectively. Putting $x=y=0$ in (3), we have

$$\mathcal{P}'_{\mu,\nu}(\phi(0,0),5040t)\leq_L \mathcal{P}_{\mu,\nu}(f(0),t) \tag{6}$$

for all $t>0$. Replacing (x,y) by $(0,x)$ in (3), we get

$$\begin{aligned}\mathcal{P}'_{\mu,\nu}(\phi(0,x),t)\leq_L \mathcal{P}_{\mu,\nu}(&f(4x)-7f(3x)+21f(2x)-35f(x)-21f(-x)\\
&+7f(-2x)-f(-3x)+35f(0)-5040f(x),t)\end{aligned} \tag{7}$$

for all $x \in X$ and $t > 0$. Interchanging (x, y) into $(x, -x)$ in 3, we obtain

$$\mathcal{P}'_{\mu,\nu}(\phi(x,-x),t) \leq_L \mathcal{P}_{\mu,\nu}(f(-3x) - 7f(-2x) + 21f(-x) - 35f(0) + 35f(x) - 21f(2x) + 7f(3x) - f(4x) - 5040f(-x), t) \tag{8}$$

for all $x \in X$ and $t > 0$. By (7) and (8), we obtain

$$\mathcal{T}(\mathcal{P}'_{\mu,\nu}(\phi(0,x),2520t), \mathcal{P}'_{\mu,\nu}(\phi(x,-x),2520t)) \leq_L \mathcal{P}_{\mu,\nu}(f(x) + f(-x), t) \tag{9}$$

for all $x \in X$ and $t > 0$. Substituting (x, y) by $(4x, x)$ in (3), we get

$$\mathcal{P}'_{\mu,\nu}(\phi(x,4x),t) \leq_L \mathcal{P}_{\mu,\nu}(f(8x) - 7f(7x) + 21f(6x) - 35f(5x) + 35f(4x) - 21f(3x) + 7f(2x) - 5041f(x), t) \tag{10}$$

for all $x \in X$ and $t > 0$. Again, replacing (x, y) by $(0, 2x)$ in (3), we arrive at

$$\mathcal{P}'_{\mu,\nu}(\phi(0,2x),t) \leq_L \mathcal{P}_{\mu,\nu}(f(8x) - 7f(6x) + 21f(4x) - 5075f(2x) + 35f(0) - 21f(-2x) + 7f(-4x) - f(-6x), t) \tag{11}$$

for all $x \in X$ and $t > 0$. It follows from (10) and (11) that

$$\mathcal{T}(\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{2}), \mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{2})) \leq_L \mathcal{P}_{\mu,\nu}(-7f(7x) + 28f(6x) - 35f(5x) + 14f(4x) - 21f(3x) + 5082f(2x) - 5041f(x) - 35f(0) + 21f(-2x) - 7f(-4x) + f(-6x), t) \tag{12}$$

for all $x \in X$ and $t > 0$. By (6), (9) and (12), we find

$$\begin{aligned}\mathcal{T}^8(&\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{10}),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{10}), \mathcal{P}'_{\mu,\nu}(\phi(0,6x),504t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),504t), \mathcal{P}'_{\mu,\nu}(\phi(0,4x),72t), \mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),72t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(0,2x),24t), \mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),24t), \mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{144}{5}t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(-7f(7x) + 27f(6x) - 35f(5x) + 21f(4x)\\ &- 21f(3x) + 5061f(2x) - 5041f(x), t)\end{aligned} \tag{13}$$

for all $x \in X$ and $t > 0$. Putting (x, y) by $(3x, x)$ in (3), we get

$$\mathcal{P}'_{\mu,\nu}(\phi(3x,x),t) \leq_L \mathcal{P}_{\mu,\nu}(f(7x) - 7f(6x) + 21f(5x) - 35f(4x) + 35f(3x) - 21f(2x) - 5033f(x) - f(0), t) \tag{14}$$

for all $x \in X$ and $t > 0$. Using (6) and (14), we have

$$\mathcal{T}(\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{2}), \mathcal{P}'_{\mu,\nu}(\phi(0,0),2520t)) \leq_L \mathcal{P}_{\mu,\nu}(f(7x) - 7f(6x) + 21f(5x) - 35f(4x) + 35f(3x) - 21f(2x) - 5033f(x), t) \tag{15}$$

for all $x \in X$ and $t > 0$. Hence

$$\begin{aligned}\mathcal{T}(\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{14}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),360t)) \leq_L\ & \mathcal{P}_{\mu,\nu}(7f(7x)-49f(6x)+147f(5x)\\ & -245f(4x)+245f(3x)-147f(2x)\\ & -35231f(x),t) \qquad (16)\end{aligned}$$

for all $x \in X$ and $t > 0$. By (13) and (16), we deduce that

$$\begin{aligned}\mathcal{T}^{10}(&\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{20}),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{20}),\mathcal{P}'_{\mu,\nu}(\phi(0,6x),252t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),252t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),36t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),36t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(0,2x),12t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),12t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{72}{5}t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{28}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),180t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(-22f(6x)+112f(5x)-224f(4x)\\ &+224f(3x)+4914f(2x)-40272f(x),t) \qquad (17)\end{aligned}$$

for all $x \in X$ and $t > 0$. Replacing (x,y) by $(2x,x)$ in (3), we obtain

$$\begin{aligned}\mathcal{P}'_{\mu,\nu}(\phi(2x,x),t) \leq_L\ & \mathcal{P}_{\mu,\nu}(f(6x)-7f(5x)+21f(4x)\\ & -35f(3x)+35f(2x)-5061f(x)+7f(0)-f(-x),t) \qquad (18)\end{aligned}$$

for all $x \in X$ and $t > 0$. Appling (6), (9) and (18), we get

$$\begin{aligned}\mathcal{T}^{3}(&\mathcal{P}'_{\mu,\nu}(\phi(0,x),840t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),840t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\frac{t}{3}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),240t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(f(6x)-7f(5x)+21f(4x)\\ &-35f(3x)+35f(2x)-5060f(x),t) \qquad (19)\end{aligned}$$

for all $x \in X$ and $t > 0$. The relation (19) implies that

$$\begin{aligned}\mathcal{T}^{3}(&\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{420}{11}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{420}{11}t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\frac{t}{66}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{120}{11}t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(22f(6x)-154f(5x)+462f(4x)\\ &-770f(3x)+770f(2x)-111320f(x),t) \qquad (20)\end{aligned}$$

for all $x \in X$ and $t > 0$. Plugging (17) to (20), one can obtain

$$\begin{aligned}\mathcal{T}^{14}(&\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{40}),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{40}),\mathcal{P}'_{\mu,\nu}(\phi(0,6x),126t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),126t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),18t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),18t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(0,2x),6t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),6t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{36}{5}t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{56}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),90t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{210}{11}t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{210}{11}t),\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\frac{t}{132}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{60}{11}t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(-21f(5x)+119f(4x)-273f(3x)\\ &+2842f(2x)-75796f(x),t) \qquad (21)\end{aligned}$$

for all $x \in X$ and $t > 0$. Letting $y = x$ in (3), we have

$$\begin{aligned}\mathcal{P}'_{\mu,\nu}(\phi(x,x),t) \leq_L\ &\mathcal{P}_{\mu,\nu}(f(5x)-7f(4x)+21f(3x)\\ &-35f(2x)-5005f(x)+7f(-x)-f(-2x)-21f(0),t) \qquad (22)\end{aligned}$$

for all $x \in X$ and $t > 0$. Using (6), (9) and (22), we get

$$\begin{aligned}\mathcal{T}^{5}(&\mathcal{P}'_{\mu,\nu}(\phi(0,x),90t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),90t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(0,2x),630t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),630t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(x,x),\frac{t}{4}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),60t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(f(5x)-7f(4x)+21f(3x)\\ &-34f(2x)-5012f(x),t) \qquad (23)\end{aligned}$$

for all $x \in X$ and $t > 0$. So,

$$\begin{aligned}\mathcal{T}^{5}(&\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{30}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{30}{7}t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(0,2x),30t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),30t),\\ &\mathcal{P}'_{\mu,\nu}(\phi(x,x),\frac{t}{84}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{20}{7}t))\\ &\leq_L \mathcal{P}_{\mu,\nu}(21f(5x)-147f(4x)+441f(3x)\\ &-714f(2x)-105252f(x),t) \qquad (24)\end{aligned}$$

for all $x \in X$ and $t > 0$. It follows (21) and (24) that

$$\begin{aligned}
\mathcal{T}^{20}(&\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{80}),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{80}),\mathcal{P}'_{\mu,\nu}(\phi(0,6x),63t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),63t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),9t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),9t)\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,2x),3t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),3t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{18}{5}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{112}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),45t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{105}{11}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{105}{11}t),\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\frac{t}{264}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{30}{11}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{15}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{15}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),15t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),15t),\mathcal{P}'_{\mu,\nu}(\phi(x,x),\frac{t}{168}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{10}{7}t))\\
&\leq_L \mathcal{P}_{\mu,\nu}(-28f(4x)+168f(3x)\\
&+2128f(2x)-181048f(x),t) \qquad (25)
\end{aligned}$$

for all $x \in X$ and $t > 0$. Replacing (x, y) by $(-x, x)$ in (3), we obtain

$$\begin{aligned}
\mathcal{P}'_{\mu,\nu}(\phi(-x,x),t) &\leq_L \mathcal{P}_{\mu,\nu}(f(3x)-7f(2x)-5019f(x)\\
&+35f(-x)-21f(-2x)+7f(-3x)-f(-4x)-35f(0),t) \qquad (26)
\end{aligned}$$

for all $x \in X$ and $t > 0$. By (6), (9) and (26), we have

$$\begin{aligned}
\mathcal{T}^{9}(&\mathcal{P}'_{\mu,\nu}(\phi(-x,x),\frac{t}{6}),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),420t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),420t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,3x),60t),\mathcal{P}'_{\mu,\nu}(\phi(3x,-3x),60t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),20t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),20t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),12t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),12t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,0),24t)) \leq_L \mathcal{P}_{\mu,\nu}(f(4x)-6f(3x)\\
&+14f(2x)-5040f(x),t) \qquad (27)
\end{aligned}$$

for all $x \in X$ and $t > 0$. Thus

$$\begin{aligned}
\mathcal{T}^{9}(&\mathcal{P}'_{\mu,\nu}(\phi(-x,x),\frac{t}{168}),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),15t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),15t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,3x),\frac{15}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(3x,-3x),\frac{15}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{5}{7}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),\frac{5}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{3}{7}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{3}{7}t),\\
&\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{6}{7}t)) \leq_L \mathcal{P}_{\mu,\nu}(28f(4x)-168f(3x)\\
&+392f(2x)-141120f(x),t) \qquad (28)
\end{aligned}$$

for all $x \in X$ and $t > 0$. By (25) and (28), we have

$$\begin{aligned}
&\mathcal{T}^{30}(\mathcal{P}'_{\mu,\nu}(\phi(4x,x),\frac{t}{160}),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{t}{160}),\mathcal{P}'_{\mu,\nu}(\phi(0,6x),\frac{63}{2}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(6x,-6x),\frac{63}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),\frac{9}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),\frac{9}{2}t)\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{3}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),\frac{3}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{9}{5}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(3x,x),\frac{t}{224}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{45}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{105}{22}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{105}{22}t),\mathcal{P}'_{\mu,\nu}(\phi(2x,x),\frac{t}{528}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{15}{11}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{15}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{15}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{15}{2}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),\frac{15}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(x,x),\frac{t}{334}),\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{5}{7}t)\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(-x,x),\frac{t}{334}),\mathcal{P}'_{\mu,\nu}(\phi(0,4x),\frac{15}{2}t),\mathcal{P}'_{\mu,\nu}(\phi(4x,-4x),\frac{15}{2}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(0,3x),\frac{15}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(3x,-3x),\frac{15}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(0,2x),\frac{5}{14}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(2x,-2x),\frac{5}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(0,x),\frac{3}{14}t),\mathcal{P}'_{\mu,\nu}(\phi(x,-x),\frac{3}{14}t),\\
&\quad\mathcal{P}'_{\mu,\nu}(\phi(0,0),\frac{6}{14}t))\\
&\quad\leq_L \mathcal{P}_{\mu,\nu}(2520f(2x)-322560f(x),t) \qquad (29)
\end{aligned}$$

for all $x \in X$ and $t > 0$. Thus

$$\Lambda(x,t) \leq_L \mathcal{P}_{\mu,\nu}(f(2x)-2^7f(x),t) \qquad (30)$$

where $\Lambda(x,t)$ is defined in (5). Thus

$$\Lambda\left(2^{\frac{1}{2}(l-1)}x, 2^{\frac{7}{2}(l+1)}t\right) \leq_L \mathcal{P}_{\mu,\nu}(2^{-7l}f(2^lx)-f(x),t) \qquad (31)$$

for all $x \in X$ and $t > 0$. We consider the set $\Omega = \{h : X \longrightarrow Y\}$ and introduce the generalized metric on X as follows:

$$d(h_1,h_2) := \inf\{C \in (0,\infty) : \Lambda\left(2^{\frac{1}{2}(l-1)}x, t\right) \leq_L \mathcal{P}_{\mu,\nu}(h_1(x)-h_2(x),Ct),\ \forall x \in X, t > 0\}.$$

if there exists a constant C, and let $d(h_1,h_2) = \infty$, otherwise. It is easy to check that d is a complete metric (see also [9]). Define the mapping $\mathcal{J} : \Omega \longrightarrow \Omega$ by $\mathcal{J}h(x) = 2^{-7l}h(2^lx)$ for all $x \in X$. Given $h_1, h_2 \in \Omega$ and ϵ be an arbitrary constant with $d(h_1,h_2) < \epsilon$. Then

$$\Lambda\left(2^{\frac{1}{2}(l-1)}x, t\right) \leq_L \mathcal{P}_{\mu,\nu}(h_1(x)-h_2(x),\epsilon t)$$

for all $x \in X$ and $t > 0$. So,

$$\Lambda\left(2^{\frac{1}{2}(l-1)}x, \frac{2^{7l}}{\alpha^l}t\right) \leq_L \mathcal{P}_{\mu,\nu}(h_1(2^l x) - h_2(2^l x), 2^{7l}\epsilon t) = \mathcal{P}_{\mu,\nu}(\mathcal{J}h_1(x) - \mathcal{J}h_2(x), \epsilon t)$$

for all $x \in X$ and $t > 0$. Hence, $d(\mathcal{J}h_1, \mathcal{J}h_2) \leq \frac{\alpha^l}{2^{7l}}d(h_1, h_2)$ for all $h_1, h_2 \in \Omega$. Thus $\mathcal{J}$ is a strictly contractive mapping of Ω with the Lipschitz constant $\frac{\alpha^l}{2^{7l}}$. It follow from (31) that $d(f, \mathcal{J}f) \leq 2^{\frac{-7}{2}(l+1)}$. By Theorem 2.1, there exists a mapping $H : X \longrightarrow Y$ satisfying:

(1) H is a unique fixed point of $\mathcal{J}$ in the set $\Omega_1 = \{h \in \omega : d(f, h) < \infty\}$, which is satisfied

$$H(2^l x) = 2^{7l}H(x) \tag{32}$$

for all $x \in X$. In other words, there exists a $C > 0$ with

$$\Lambda\left(2^{\frac{1}{2}(l-1)}x, t\right) \leq_L \mathcal{P}_{\mu,\nu}(f(x) - H(x), Ct)$$

for all $x \in X$ and $t > 0$.

(2) $d(\mathcal{J}^n f, H) \to 0$ as $n \to \infty$. This implies that

$$H(x) = \lim_{n\to\infty} \mathcal{J}^n f(x) = \lim_{n\to\infty} 2^{-7ln} f(2^{ln}x)$$

for all $x \in X$.

(3) For every $f \in \Omega$, we have $d(f, H) \leq \frac{1}{1-\frac{\alpha^l}{2^{7l}}}d(\mathcal{J}f, f)$. Since, $d(\mathcal{J}f, f) \leq 2^{\frac{-7}{2}(l+1)}$, we have $d(f, H) \leq \frac{\alpha^{-\frac{1}{2}(l-1)}}{|128-\alpha|}$. The last inequality shows that

$$\Lambda(x, t) \leq_L \mathcal{P}_{\mu,\nu}\left(f(x) - H(x), \frac{\alpha^{-\frac{1}{2}(l-1)}}{|128 - \alpha|}t\right) \tag{33}$$

for all $x \in X$ and $t > 0$. it follows from the relations (2) and (33) that the inequality (4) holds. Replacing $2^{ln}x$ and $2^{ln}y$ by x and y in (3), respectively, we get

$$\mathcal{P}'_{\mu,\nu}\left(\phi(x, y), \frac{2^{7n}}{\alpha^n}t\right) \leq_L \mathcal{P}'_{\mu,\nu}(D_h f(2^{ln}x, 2^{ln}y), 2^{7n}t) \leq_L \mathcal{P}_{\mu,\nu}\left(2^{-7ln}D_h f(2^{ln}x, 2^{ln}y), t\right)$$

for all $x, y \in X$ and $t > 0$. Letting n tends to infinity, we see that H is a heptic mapping. □

The following corollaries are the direct consequences of Theorem 3.2 concerning the stability of (1).

Corollary 3.3. *Let λ be a nonnegative real number with $\lambda \neq 7$, X be a normed space with norm $\|\cdot\|$, $(Z, \mathcal{P}'_{\mu,\nu}, \mathcal{T})$ be an intuitionistic fuzzy normed space, $(Y, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ be a complete intuitionistic fuzzy normed space, and let $z_0 \in Z$. If $f : X \longrightarrow Y$ is a mapping such that*

$$\mathcal{P}'_{\mu,\nu}((\|x\|^\lambda + \|y\|^\lambda)z_0, t) \leq_L \mathcal{P}_{\mu,\nu}(D_h f(x, y), t)$$

for all $x, y \in X$ and $t > 0$, then there exists a unique heptic mapping $H : X \longrightarrow Y$ such that

$$\mathcal{P}'_{\mu,\nu}(\|x\|^\lambda z_0, \min\{\frac{63}{4(4^\lambda+1)}, \frac{79380}{2\cdot 6^\lambda}, \frac{11340}{2\cdot 4^\lambda}, \frac{45}{4(3^\lambda+1)}, \frac{105}{22(2^\lambda+1)}, \frac{9540}{2\cdot 3^\lambda}\}|128-2^\lambda|t) \\ \leq_L \mathcal{P}_{\mu,\nu}(H(x)-f(x),t)$$

for all $x \in X$ and $t > 0$.

Proof. Setting $\phi(x,y) := (\|x\|^\lambda + \|y\|^\lambda)z_0$ and applying Theorem 3.2, we get the desired result. $\square$

The proof of the following result is similar to the above corollary. So, it is omitted.

Corollary 3.4. *Let λ be a nonnegative real number with $\lambda := r + s \neq 7$, X be a normed space with norm $\|\cdot\|$, $(Z, \mathcal{P}'_{\mu,\nu}, \mathcal{T})$ be an intuitionistic fuzzy normed space, $(Y, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ be a complete intuitionistic fuzzy normed space, and let $z_0 \in Z$. If $f : X \longrightarrow Y$ is a mapping such that*

$$\mathcal{P}'_{\mu,\nu}((\|x\|^r\|y\|^s + \|x\|^\lambda + \|y\|^\lambda)z_0, t) \leq_L \mathcal{P}_{\mu,\nu}(D_hf(x,y),t)$$

for all $x, y \in X$ and $t > 0$, then there exists a unique heptic mapping $H : X \longrightarrow Y$ such that

$$\mathcal{P}'_{\mu,\nu}(\|x\|^\lambda z_0, \min\{\frac{63}{4(4^r+4^\lambda+1)}, \frac{79380}{3\cdot 6^\lambda}, \frac{11340}{3\cdot 4^\lambda}, \frac{45}{4(3^r+3^\lambda+1)}, \frac{105}{22(2^r+2^\lambda+1)}, \frac{9540}{3\cdot 3^\lambda}\} \\ |128-2^\lambda|t) \leq_L \mathcal{P}_{\mu,\nu}(H(x)-f(x),t)$$

for all $x \in X$ and $t > 0$.

The idea of th following example is taken from [31, Example 3.7] which provides an illustration.

Example 3.1. [31] Let $(A, \|\cdot\|)$ be a Banach algebra. Then $(A, \mathcal{P}_{\mu,\nu}, \mathcal{T})$ is an IFN-space for which $\mathcal{P}_{\mu,\nu}$ is the intuitionistic fuzzy set defined in Example 2.5. Define $f : A \longrightarrow A$ via $f(x) = x^7 + \|x\|x_0$, where x_0 is a unit vector in A. An easy computation shows that

$$\|D_hf(x,y)\| \leq 128\|x\| + 5177\|y\|$$

for all $x, y \in A$. Thus

$$\mathcal{P}_{\mu,\nu}([128\|x\| + 5177\|y\|]x_0, t) \leq_L \mathcal{P}_{\mu,\nu}(D_hf(x,y),t)$$

for all $x, y \in A$ and $t > 0$. Consider $\phi : A \times A \longrightarrow A$ defined through

$$\phi(x,y) = (128\|x\| + 5177\|y\|)x_0 \qquad (x, y \in A).$$

We have

$$\mathcal{P}_{\mu,\nu}(2^l\phi(x,y),t) \leq_L \mathcal{P}_{\mu,\nu}(\phi(2^lx, 2^ly),t)$$

for all $x, y \in A$ and $t > 0$ in which $l \in \{1, -1\}$. Therefore, all the conditions of Theorem 3.2 hold when $\alpha = 2 \neq 2^7$. It implies that f can be approximated by a heptic mapping. In fact there exists a unique heptic mapping $H : X \longrightarrow Y$ such that

$$\mathcal{P}'_{\mu,\nu}(\|x\|x_0, \frac{2205}{56947}t) \leq_L \mathcal{P}_{\mu,\nu}(H(x) - f(x), t)$$

for all $x, y \in A$ and $t > 0$.

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References

[1] M. Arunkumar, A. Bodaghi, T. Namachivayam and E. Sathya, *A new type of the additive functional equations on intuitionistic fuzzy normed spaces*, Commun. Korean Math. Soc., **32**(4)(2017), 915-932.

[2] T. Bag and S. K. Samanta, *Some fixed point theorems on fuzzy normed linear spaces*, Info Sci., **177**(2007), 3271-3289.

[3] T. Bag and S. K. Samanta, *Fuzzy bounded linear operators*, Fuzzy Sets Syst., **151**(2005), 513-547.

[4] A. Bodaghi, *Intuitionistic fuzzy stability of the generalized forms of cubic and quartic functional equations*, J. Intel. Fuzzy Syst., **30**(2016), 2309-2317.

[5] A. Bodaghi, I. A. Alias and M. Eshaghi Gordji, *On the stability of quadratic double centralizers and quadratic multipliers: A fixed point approach,* J. Inequal. Appl., **2011**(2011), Article ID 957541.

[6] A. Bodaghi, I. A. Alias and M. H. Ghahramani, *Ulam stability of a quartic functional equation*, Abs. Appl. Anal., **2012**(2012), Art. ID 232630.

[7] A. Bodaghi, S. M. Moosavi and H. Rahimi, *The generalized cubic functional equation and the stability of cubic Jordan ∗-derivations*, Ann. Univ. Ferrara., **59**(2013), 235-250.

[8] A. Bodaghi, C. Park and J. M. Rassias, *Fundamental stabilities of the nonic functional equation in intuitionistic fuzzy normed spaces*, Commun. Korean Math. Soc., **31**(4)(2016), 729-743.

[9] L. Cădariu and V. Radu, *Fixed points and stability for functional equations in probabilistic metric and random normed spaces*, Fixed Point Theory Appl., **2009**(2009), Article ID 589143,1 8 pages.

[10] L. Cădariu and V. Radu, *Fixed points and the stability of quadratic functional equations*, An. Univ. Timişoara, Ser. Mat. Inf., **41**(2003), 25-48.

[11] L. Cădariu and V. Radu, *On the stability of the Cauchy functional equation: A fixed point approach*, Grazer Math. Ber., **346**(2004), 43-52.

[12] J. B. Diaz and B. Margolis, *A fixed point theorem of the alternative for contractions on a generalized complete metric space,* Bull. Amer. Math. Soc., **74**(1968), 305-309.

[13] J. X. Fang, *On I-topology generated by fuzzy norm*, Fuzzy Sets Syst., **157**(2006), 2739-2750.

[14] D. H. Hyers, *On the stability of the linear functional equation,* Proc. Nat. Acad. Sci. USA, **27**(1941), 222-224.

[15] M. Eshaghi Gordji and A. Bodaghi,*On the Hyers-Ulam-Rassias stability problem for quadratic functional equations*, East J. App., **16**(2)(2010), 123-130.

[16] M. Eshaghi Gordji, A. Bodaghi and C. Park, *A fixed point approach to the stability of double Jordan centralizers and Jordan multipliers on Banach algebras*, U.P.B. Sci. Bull., Series A., **73**(2)(2011), 65-73.

[17] S. A. Mohiuddine and H. Sevli, *Stability of Pexiderized quadratic functional equation in intuitionistic fuzzy normed space*, J. Comp. Appl. Math., **235**(2011), 2137-214.

[18] M. Mursaleen and S. A. Mohiuddine, *On stability of a cubic functional equation in intuitionistic fuzzy normed spaces*, Chaos, Solitons Fract., **42**(2009), 2997-3005.

[19] M. Mursaleen and Q. M. D. Lohani, *Intuitionistic fuzzy 2-normed space and some related concepts*, Chaos, Solitons Fract., **42**(2009), 224-234.

[20] J. H. Park, *Intuitionistic fuzzy metric spaces*, Chaos, Solitons Fract., **22**(2004), 1039-1046.

[21] R. Saadati, *A note on some results on the IF-normed spaces*, Chaos, Solitons Fract., **41**(2009), 206-213.

[22] R. Saadati, Y. J. Cho and J. Vahidi, *The stability of the quartic functional equation in various spaces*, Comp. Math. Appl., **60**(2010), 1994-2002.

[23] R. Saadati and C. Park, *Non-archimedean L-fuzzy normed spaces and stability of functional equations*, Comp. Math. Appl., **60**(2010), 2488-2496.

[24] R. Saadati and J. H. Park, *On the intuitionistic fuzzy topological spaces*, Chaos, Solitons Fract., **27**(2006), 331-344.

[25] R. Saadati, A. Razani and H. Adibi, *A common fixed point theorem in $\mathcal{L}$-fuzzy metric spaces*, Chaos, Solitons Fract., **33**(2007), 354-363.

[26] R. Saadati, S. Sedghi and N. Shobe, *Modified intuitionistic fuzzy metric spaces and some fixed point theorems*, Chaos, Solitons Fract., **38**(2008), 36-47.

[27] S. M. Ulam, *Problems in modern mathematics,* Chapter VI, Science Ed., Wiley, New York, 1940.

[28] T. Z. Xu, J. M. Rassias and W. X. Xu, *A generalized mixed quadratic-quartic functional equation*, Bull. Malay. Math. Sci. Soc., **35**(3)(2012), 633-649.

[29] T. Z. Xu, M. J. Rassias and W. X. Xu, *Stability of a general mixed additive-cubic functional equation in non-archimedean fuzzy normed spaces*, J. Math. Phys., **51**(2010), 093508, 19 pages.

[30] T. Z. Xu and J. M. Rassias, *Approximate septic and octic mappings in quasi-β-normed spaces*, J. Comp. Anal. Appl., **15**(6)(2013), 1110-1119.

[31] T. Z. Xu, M. J. Rassias, W. X. Xu and J. M. Rassias, *A fixed point approach to the intuitionistic fuzzy stability of quintic and sextic functional equations*, Iran. J. Fuzzy Syst., **9**(5)(2012), 21-40.

[32] L. A. Zadeh, *Fuzzy sets*, Inf. Cont., **8**(1965), 338-353.

Department of Mathematics, South Tehran Branch, Islamic Azad University, Tehran, Iran
mshafieemousavi@azad.ac.ir

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