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# Stability of two multi-quadratic mappings by a fixed point method

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Abstract. In this paper, the Găvruţa stability of two multi-quadratic functional equations are established by a known fixed point theorem. As an example, the Hyers-Ulam, Rassias stability and hyperstability of the mentioned mappings are proved in the setting of Banach spaces.

## 1. Introduction

In 1940, Ulam [**22**] asked the question concerning the stability of group homomorphisms. The famous Ulam stability problem was partially solved by Hyers [**13**] for the linear functional equation of Banach spaces. Hyers' theorem was generalized by Aoki [**1**] for additive mappings and by Th. M. Rassias [**18**] for linear mappings by considering an unbounded Cauchy difference. A generalization of the Rassias theorem was obtained by Găvruţa [**12**] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias approach. Recall that an equation is *stable* in some class of functions if any function from that class, satisfying the equation approximately (in some sense), is near (in some way) to an exact solution of the equation. A lot of information about various functional equations can be found for instance in papers and books [**2, 3, 5, 14, 16, 17, 19**] and also references therein.

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The stability problem for quadratic functional equation

$$Q(x+y)+Q(x-y)=2Q(x)+2Q(y) \tag{1}$$

has been studied in normed spaces by Skof [20] with a constant bound. Thereafter, Czerwik [10] proved the Hyers-Ulam stability of the quadratic functional equation with a nonconstant bound. Some different versions of quadratic functional equation (1) and its stabilities with applications are available in [4, 11] and other resources.

A general form of (1), say the *generalized quadratic* functional equation is as follows:

$$\mathfrak{Q}(ax+y)+\mathfrak{Q}(ax-y)=2a^2\mathfrak{Q}(x)+2\mathfrak{Q}(y), \tag{2}$$

where $a$ is a fixed non-zero number in $\mathbb{Q}$. Moreover, the different form of a quadratic functional equation was presented in [15] as follows:

$$Q(ax+y)+Q(ax-y)=Q(x+y)+Q(x-y)+2\left(a^2-1\right)Q(x), \tag{3}$$

where, $a$ is a fixed integer with $a\neq 0,\pm 1$.

Let $V$ be a commutative group, $W$ be a linear space, and $n\geq 2$ be an integer. For the set $X$, we denote $\overbrace{X\times X\times\cdots\times X}^{n-\text{times}}$ by $X^n$. For any $l\in\mathbb{N}_0, n\in\mathbb{N}$, $t=(t_1,\cdots,t_n)\in\{-1,1\}^n$ and $x=(x_1,\cdots,x_n)\in V^n$ we write $lx:=(lx_1,\cdots,lx_n)$ and $tx:=(t_1x_1,\cdots,t_nx_n)$. Recall that a mapping $f:V^n\longrightarrow W$ is called *multi-quadratic* if it is quadratic (satisfying quadratic functional equation (1)) in each component. It is shown in [23] that the system of functional equations defining a multi-quadratic mappings can be unified as a single equation. Indeed, Zhao et al. proved that the mapping $f:V^n\longrightarrow W$ is multi-quadratic if and only if the relation

$$\sum_{s\in\{-1,1\}^n} f(x_1+sx_2)=2^n\sum_{j_1,j_2,\ldots,j_n\in\{1,2\}} f(x_{1j_1},x_{2j_2},\ldots,x_{nj_n})$$

holds, where $x_j=(x_{1j},x_{2j},\ldots,x_{nj})\in V^n$ with $j\in\{1,2\}$. Various versions of multi-quadratic mappings were introduced and studied for example in [6], [9] and [21].

In this paper, we establish the Găvruţa stability of multi-quadratic functional equations (taken from (2) and (3) introduced in [6] and [15]) by a known fixed point theorem. As some corollaries, we prove the Hyers-Ulam, Rassias stability and hyperstability of the mentioned mappings in the setting of Banach spaces.

## 2. Preliminary notations

We commence this section with the following definition which was defined in [6] and [15]. Throughout this section, assume that $V$ and $W$ are vector spaces over $\mathbb{Q}$ (the rationals).

**Definition 2.1.** Let $n \in \mathbb{N}$. Suppose that a mapping $f : V^n \longrightarrow W$ is given. Then

(i) (see [6].) $f$ is called *$n$-quadratic* or *multi-quadratic* (the first kind) if $f$ is quadratic in each variable (see equation (2));

(ii) (see [15].) $f$ is called *$n$-quadratic* or *multi-quadratic* (the second kind) if $f$ is quadratic in each variable (see equation (3)).

Let $n \in \mathbb{N}$ and $x_i^n = (x_{i1}, x_{i2}, \dots, x_{in}) \in V^n$, where $i \in \{1, 2\}$. Let $l_j \in \{1, 2\}$. Put

$$M_i^n = \{x = (x_{l_1 1}, x_{l_2 2}, \dots, x_{l_n n}) \in V^n | \operatorname{Card}\{l_j : l_j = 1\} = i\}. \tag{4}$$

We shall denote $x_i^n$ and $M_i^n$ by $x_i$ and $M_i$, respectively if there is no risk of ambiguity. Let $x_1, x_2 \in V^n$ and $k \in \mathbb{N}_0$ with $0 \le k \le n$. Put

$$\mathcal{A} = \{\mathfrak{A}_n = (A_1, \dots, A_n) |\ A_j \in \{x_{1j} \pm x_{2j}, x_{1j}\}\},$$

where $j \in \{1, \dots, n\}$. Consider $\mathcal{A}_k^n := \{\mathfrak{A}_n \in \mathcal{A} |\ \operatorname{Card}\{A_j : A_j = x_{1j}\} = k\}$. For a multi-quadratic mapping (the second kind) $f : V^n \longrightarrow W$ we use the following notation:

$$f(\mathcal{A}_k^n) := \sum_{\mathfrak{A}_n \in \mathcal{A}_k^n} f(\mathfrak{A}_n), \tag{5}$$

Let $a$ be fixed non-zero number in $\mathbb{Q}$. Recall that a mapping $f : V^n \longrightarrow W$ satisfies (has) the *quartic condition* in the $j$th variable if

$$f(x_1, \dots, x_{j-1}, ax_j, x_{j+1}, \dots, x_n) = a^2 f(x_1, \dots, x_{j-1}, x_j, x_{j+1}, \dots, x_n),$$

for all $x_1, \dots, x_n \in V$ and for all $j \in \{1, \dots, n\}$. The following result was proved in [6, Theorem 2.2].

**Theorem 2.1.** *Consider the mapping $f : V^n \longrightarrow W$. Then, the following assertions are equivalent:*

(i) *$f$ is multi-quadratic (the first kind);*

(ii) *$f$ satisfies equation*

$$\sum_{q \in \{-1,1\}^n} f(ax_1 + qx_2) = 2^n \sum_{i=0}^{n} a^{2i} \sum_{x \in M_i} f(x), \tag{6}$$

*with the quadratic condition in all variables, where $M_i$ is defined in (4) and $a$ is a fixed non-zero number in $\mathbb{Q}$.*

The following result was proved in [15, Theorem 3.3].

**Proposition 2.2.** *Consider the mapping $f : V^n \longrightarrow W$. Then, the following assertions are equivalent:*

(i) *$f$ is multi-quadratic (the second kind);*

(ii) *$f$ satisfies equation*

$$\sum_{q\in\{-1,1\}^n} f(ax_1+qx_2)=\sum_{k=0}^{n}\left(2m^2-2\right)^k f\left(\mathcal{A}_k^n\right), \tag{7}$$

*with the quadratic condition in each variable, where $f\left(\mathcal{A}_k^n\right)$ is defined in (5) and $a$ is a fixed integer with $a\neq 0,\pm1$.*

A direct consequence of Theorem 2.1 and Proposition 2.2 is indicated as follows:

**Corollary 2.3.** *Suppose that mapping $f: V^n\longrightarrow W$ is given with the quadratic condition in each variable. Then, $f$ satisfies equation (6) if and only if it fulfills equation (7).*

## 3. Găvruţa and Rassias Stability results

In this section, we study the various stabilities of equations (6) and (7). From now on, for two sets $X$ and $Y$, the set of all mappings from $X$ to $Y$ is denoted by $Y^X$.Here, we indicate the following theorem which is a fundamental result in fixed point theory [7, Theorem 1]. This result plays a key tool in obtaining our objective in this section.

**Theorem 3.1.** *Given the hypotheses*

(A1) *$Y$ is a Banach space, $\mathcal{S}$ is a nonempty set, $j\in\mathbb{N}$, $g_1,...,g_j:\mathcal{S}\longrightarrow\mathcal{S}$ and $L_1,...,L_j:\mathcal{S}\longrightarrow\mathbb{R}_+$,*

(A2) *$\mathcal{T}: Y^{\mathcal{S}}\longrightarrow Y^{\mathcal{S}}$ is an operator satisfying the inequality*

$$\|\mathcal{T}\lambda(x)-\mathcal{T}\mu(x)\|\leq\sum_{i=1}^{j}L_i(x)\left\|\lambda(g_i(x))-\mu(g_i(x))\right\|, \quad \lambda,\mu\in Y^{\mathcal{S}}, x\in\mathcal{S},$$

(A3) *$\Lambda:\mathbb{R}_+^{\mathcal{S}}\longrightarrow\mathbb{R}_+^{\mathcal{S}}$ is an operator defined through*

$$\Lambda\delta(x):=\sum_{i=1}^{j}L_i(x)\delta(g_i(x)) \qquad \delta\in\mathbb{R}_+^{\mathcal{S}}, x\in\mathcal{S}.$$

*Suppose that a function $\theta:\mathcal{S}\longrightarrow\mathbb{R}_+$ and a mapping $\phi:\mathcal{S}\longrightarrow Y$ fulfill the next two properties.*

$$\|\mathcal{T}\phi(x)-\phi(x)\|\leq\theta(x), \quad \theta^*(x):=\sum_{l=0}^{\infty}\Lambda^l\theta(x)<\infty \qquad (x\in\mathcal{S}).$$

*Then, there exists a unique fixed point $\psi$ of $\mathcal{T}$ such that*

$$\|\phi(x)-\psi(x)\|\leq\theta^*(x) \qquad (x\in\mathcal{S}).$$

*Moreover, $\psi(x)=\lim_{l\to\infty}\mathcal{T}^l\phi(x)$ for all $x\in\mathcal{S}$.*

For a mapping $f : V^n \longrightarrow W$, we define two operators $\mathfrak{D}_1 f$ and $\mathfrak{D}_2 f$ from $V^n \times V^n$ into $W$ via

$$\mathfrak{D}_1 f(x_1, x_2) := \sum_{q\in\{-1,1\}^n} f(ax_1 + qx_2) - 2^n \sum_{i=0}^{n} a^{2i} \sum_{x\in M_i} f(x),$$

and

$$\mathfrak{D}_2 f(x_1, x_2) := \sum_{q\in\{-1,1\}^n} f(ax_1 + qx_2) - \sum_{k=0}^{n} \left(2a^2 - 2\right)^k f\left(\mathcal{A}_k^n\right)$$

where $M_i$ and $f\left(\mathcal{A}_k^n\right)$ are defined in (4) and (5), respectively in which $a$ is a fixed integer with $a \neq 0, \pm 1$.

We say a mapping $f : V^n \longrightarrow W$ has *zero condition* or *zero functional equation* if $f(v) = 0$ for any $v \in V^n$ with at least one component which is equal to zero. With notations above, we have the following Găvruţa stability for functional equations (6) and (7).

**Theorem 3.2.** *Let $j \in \{-1, 1\}$, $V$ be a linear space and $W$ be a Banach space. Suppose that $\phi : V^n \times V^n \longrightarrow \mathbb{R}_+$ is a mapping satisfying*

$$\lim_{l\to\infty} \left(\frac{1}{a^{2nj}}\right)^l \phi(a^{jl}x_1, a^{jl}x_2) = 0, \tag{8}$$

*for all $x_1, x_2 \in V^n$ and*

$$\Phi(x) = \frac{1}{2^n a^{n(j+1)}} \sum_{l=0}^{\infty} \left(\frac{1}{a^{2nj}}\right)^l \phi\left(a^{jl+\frac{j-1}{2}}x, 0\right) < \infty, \tag{9}$$

*for all $x \in V^n$. If $f : V^n \longrightarrow W$ is a mapping with zero condition fulfilling the inequality*

$$\|\mathfrak{D}_1 f(x_1, x_2)\| \leq \phi(x_1, x_2), \tag{10}$$

*for all $x_1, x_2 \in V^n$, then there exists a solution $\mathcal{Q} : V^n \longrightarrow W$ of (6) such that*

$$\|f(x) - \mathcal{Q}(x)\| \leq \Phi(x), \tag{11}$$

*for all $x \in V^n$. If $\mathcal{Q}$ has the quadratic condition in each of variable, then it is unique multi-quadratic.*

Proof. Putting $x = x_1$ and $x_2 = 0$ in (10), we get

$$\left\|2^n f(ax) - 2^n a^{2n} f(x)\right\| \leq \phi(x, 0), \tag{12}$$

for all $x \in V^n$ (here and the rest of the proof). Inequality (12) can be rewritten as follows:

$$\begin{cases} \left\| \frac{f(ax)}{a^{2n}} - f(x) \right\| \le \frac{1}{2^n a^{2n}} \phi(x,0), \\ \\ \| a^{2n} f(ax) - f(x) \| \le \frac{1}{2^n} \phi(x,0). \end{cases}$$

Set $\xi(x) := \frac{1}{2^n a^{n(j+1)}} \phi\left( a^{\frac{j-1}{2}} x, 0 \right)$ and $\mathcal{T}\xi(x) := \frac{1}{a^{2nj}} \xi(a^j x)$, where $\xi \in W^{V^n}$. A modification of (12) shows that $\|f(x) - \mathcal{T}f(x)\| \le \xi(x)$. Define $\Lambda\eta(x) := \frac{1}{a^{2nj}} \eta(a^j x)$ for all $\eta \in \mathbb{R}_+^{V^n}$. Considering $\mathcal{S} = V^n$, $g_1(x) = a^j x$ and $L_1(x) = \frac{1}{a^{2nj}}$ in (A3), we find that $\Lambda$ has the formation in (A3). On the other hand, we obtain

$$\|\mathcal{T}\lambda(x) - \mathcal{T}\mu(x)\| = \left\| \frac{1}{a^{2nj}} \left[ \lambda(a^j x) - \mu(a^j x) \right] \right\| \le L_1(x) \left\| \lambda(g_1(x)) - \mu(g_1(x)) \right\|.$$

for all $\lambda, \mu \in W^{V^n}$. The relation above leads us to validity of hypothesis (A2) for $\mathcal{T}$. It is easily verified that by induction on $l \in \mathbb{N}_0$ that

$$\Lambda^l \xi(x) := \left( \frac{1}{2^n a^{2nj}} \right)^l \xi(a^{jl} x) = \frac{1}{a^{n(j+1)}} \left( \frac{1}{a^{2nj}} \right)^l \phi\left( a^{jl + \frac{j-1}{2}} x, 0 \right). \tag{13}$$

It now follows that all assumptions of Theorem 3.1 are satisfied by applying (9) and (13) and thus there exists a mapping $\mathcal{Q} : V^n \longrightarrow W$ such that

$$\mathcal{Q}(x) = \lim_{l \to \infty} (\mathcal{T}^l f)(x) = \frac{1}{a^{2nj}} \mathcal{Q}(a^j x)$$

and (11) holds as well. We claim that the following inequality is true for each $x_1, x_2 \in V^n$ and $l \in \mathbb{N}_0$.

$$\|\mathfrak{D}_1(\mathcal{T}^l f)(x_1, x_2)\| \le \left( \frac{1}{a^{2nj}} \right)^l \phi(a^{jl} x_1, a^{jl} x_2). \tag{14}$$

The argument is based on induction. Inequality (10) shows that (14) is true for $l = 0$. Assume that (14) is valid for an $l \in \mathbb{N}_0$. We have

$$\begin{aligned} &\|\mathfrak{D}_1(\mathcal{T}^{l+1} f)(x_1, x_2)\| \\ &= \left\| \sum_{q \in \{-1,1\}^n} (\mathcal{T}^{l+1} f)(a x_1 + q x_2) - 2^n \sum_{i=0}^n a^{2i} \sum_{x \in M_i} (\mathcal{T}^{l+1} f)(x) \right\| \\ &= \frac{1}{a^{2nj}} \left\| \sum_{q \in \{-1,1\}^n} (\mathcal{T}^{l+1} f)(a^j(a x_1 + q x_2)) - 2^n \sum_{i=0}^n a^{2i} \sum_{x \in M_i} (\mathcal{T}^{l+1} f)(a^j x) \right\| \\ &= \frac{1}{a^{2nj}} \left\| \mathfrak{D}_1(\mathcal{T}^l f)(a^j x_1, a^j x_2) \right\| \le \left( \frac{1}{a^{2nj}} \right)^{l+1} \phi(a^{j(l+1)} x_1, a^{j(l+1)} x_2), \end{aligned}$$

for all $x_1, x_2 \in V^n$. Letting $l \to \infty$ in (14) and using (8), we reach to $\mathfrak{D}_1 \mathcal{Q}(x_1, x_2) = 0$ for all $x_1, x_2 \in V^n$. This means that the mapping $\mathcal{Q}$ satisfies (6). If $\mathcal{Q}$ has the quadratic condition in each of variable, then it is a multi-quadratic mapping by

Theorem 2.1. Finally, assume that $\mathcal{Q}': V^n \longrightarrow W$ is another multi-quadratic mapping satisfying the equation (6) and inequality (11), and fix $x \in V^n$, $l \in \mathbb{N}$. Then

$$\begin{aligned}\|\mathcal{Q}(x)-\mathcal{Q}'(x)\| &= \left\|\frac{1}{a^{2njl}}\mathcal{Q}(a^{jl}x)-\frac{1}{a^{2njl}}\mathcal{Q}'(a^{jl}x)\right\| \\ &\leq \frac{1}{a^{2njl}}\left(\|\mathcal{Q}(2^{jl}x)-f(a^{jl}x)\|+\|\mathcal{Q}'(a^{jl}x)-f(a^{jl}x)\|\right) \\ &\leq \frac{2}{a^{2njl}}\Phi(a^{jl}x) \\ &\leq \frac{1}{2^{n-1}a^{n(j+1)}}\sum_{k=l}^{\infty}\left(\frac{1}{a^{2nj}}\right)^k \phi\left(a^{kl+\frac{j-1}{2}}x,0\right).\end{aligned}$$

Now, letting $l$ to infinity and applying the convergency of series (9), we arrive that $\mathcal{Q}(x)=\mathcal{Q}'(x)$, which completes the proof. □

Let $A$ be a nonempty set, $(X,d)$ a metric space, $\psi \in \mathbb{R}_+^{A^n}$, and $\mathcal{F}_1, \mathcal{F}_2$ operators mapping a nonempty set $D \subset X^A$ into $X^{A^n}$. We say that operator equation

$$\mathcal{F}_1\varphi(a_1,\ldots,a_n)=\mathcal{F}_2\varphi(a_1,\ldots,a_n) \tag{15}$$

is $\psi$-hyperstable provided every $\varphi_0 \in D$ satisfying inequality

$$d(\mathcal{F}_1\varphi_0(a_1,\ldots,a_n),\mathcal{F}_2\varphi_0(a_1,\ldots,a_n)) \leq \psi(a_1,\ldots,a_n), \qquad a_1,\ldots,a_n \in A$$

fulfills (15); this definition is introduced in [8]. In other words, a functional equation $\mathcal{F}$ is *hyperstable* if any mapping $f$ satisfying the equation $\mathcal{F}$ approximately is a true solution of $\mathcal{F}$.

In the incoming example, we show that equation (6) is stable and hyperstable.

**Example 3.1.** Let $\delta$ and $\varepsilon$ be non-negative real numbers. Suppose that $\alpha, \alpha_{ij} > 0$ for $i \in \{1,2\}$ and $j \in \{1,\ldots,n\}$ fulfill $\sum_{i=1}^2\sum_{j=1}^n \alpha_{ij} \neq 2n$ and $\alpha \neq 2n$. Let $V$ be a normed space and $W$ be a Banach space. If $f: V^n \longrightarrow W$ is a mapping with zero condition satisfying the inequality

$$\|\mathfrak{D}_1 f(x_1,x_2)\| \leq \sum_{i=1}^{2}\sum_{j=1}^{n}\|x_{ij}\|^{\alpha}\varepsilon + \prod_{i=1}^{2}\prod_{j=1}^{n}\|x_{ij}\|^{\alpha_{ij}}\delta,$$

for all $x_1, x_2 \in V^n$. Then, there exists a solution $\mathcal{Q}: V^n \longrightarrow W$ of (6) such that

$$\|f(x)-\mathcal{Q}(x)\| \leq \frac{\delta}{2^n|a^{2n}-a^{\alpha}|}\sum_{j=1}^{n}\|x_{1j}\|^{\alpha}.$$

In particular, if $\mathcal{Q}$ has the quadratic condition in each of variable, then it is a unique multi-quadratic mapping. In the case that $\varepsilon = 0$, then $f$ is multi-quadratic. If moreover

$$\|\mathfrak{D}_1 f(x_1,x_2)\| \leq \varepsilon,$$

for all $x_1, x_2 \in V^n$, then there exists a solution $\mathcal{Q} : V^n \longrightarrow W$ of (6) such that

$$\|f(x) - \mathcal{Q}(x)\| \leq \frac{\varepsilon}{2^n(a^{2n} - 1)},$$

for all $x \in V^n$.

We have the next stability result regarding equation (7) which is analogous to Theorem 3.2 without zero condition. Since the proof is similar, we include only some parts.

**Theorem 3.3.** *Let $j \in \{-1, 1\}$, $V$ be a linear space and $W$ be a Banach space. Suppose that $\phi : V^n \times V^n \longrightarrow \mathbb{R}_+$ is a mapping satisfying*

$$\lim_{l\to\infty} \left(\frac{1}{a^{2nj}}\right)^l \phi(a^{jl}x_1, a^{jl}x_2) = 0,$$

*for all $x_1, x_2 \in V^n$. If $f : V^n \longrightarrow W$ is a mapping satisfies the inequality*

$$\|\mathfrak{D}_2 f(x_1, x_2)\| \leq \phi(x_1, x_2),$$

*for all $x_1, x_2 \in V^n$, then there exists a solution $\mathcal{Q} : V^n \longrightarrow W$ of (7) such that*

$$\|f(x) - \mathcal{Q}(x)\| \leq \Phi(x)$$

*for all $x \in V^n$, where*

$$\Phi(x) = \frac{1}{a^{n(j+1)}} \sum_{l=0}^{\infty} \left(\frac{1}{a^{2nj}}\right)^l \phi\left(a^{jl+\frac{j-1}{2}}x, 0\right) < \infty.$$

*If $\mathcal{Q}$ has the quadratic condition in each of variable, then it is unique multi-quadratic.*

Proof. Putting $x = x_1$ and $x_2 = 0$ in (10), we get

$$\left\|2^n f(ax) - \left(\sum_{k=0}^{n} \binom{n}{k} 2^{n-k}(2a^2 - 2)^k\right) f(x)\right\| \leq \phi(x, 0), \tag{16}$$

for all $x \in V^n$ and $t > 0$. An easy computation shows that

$$\sum_{k=0}^{n} \binom{n}{k} 2^{n-k}(2m^2 - 2)^k = (2m^2)^n. \tag{17}$$

It follows from (16) and (17) that

$$\left\|2^n f(ax) - 2^n a^{2n} f(x)\right\| \leq \phi(x, 0).$$

for all $x \in V^n$. The proof of Theorem 3.2 can be repeated to finalize this proof. □

According to Theorem 3.3, we observe that Example 3.1 has the similar results for equation (7) as a directed result of the mentioned theorem.

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## References

[1] T. Aoki, *On the stability of the linear transformation in Banach spaces*, J. Math. Soc. Japan., **2** (1950), 64–66.

[2] M. Arunkumar, A. Bodaghi, J. M. Rassias, and E. Sathya, *The general solution and approximations of a decic type functional equation in various normed spaces,* J. Chung. Math. Soc., **29**(1) (2016), 287–328.

[3] A. Bodaghi, *Stability of a mixed type additive and quartic functional equation*, Filomat, **28** (2014), 1629–1640.

[4] A. Bodaghi and I. A. Alias, *Approximate ternary quadratic derivations on ternary Banach algebras and $C^*$-ternary rings*, Adv. Diff. Equa., **2012**(1) (2012), 9 pages.

[5] A. Bodaghi, I. A. Alias, and M. H. Ghahramani, *Approximately cubic functional equations and cubic multipliers*, J. Inequ. Appl., **2011** (2011), Paper No. 53.

[6] A. Bodaghi, C. Park, and S. Yun, *Almost multi-quadratic mappings in non-Archimedean spaces*, AIMS Math., **5**(5) (2020), 5230–5239.

[7] J. Brzdęk, J. Chudziak, and Zs. Páles, *A fixed point approach to stability of functional equations*, Nonlinear Anal., **74** (2011), 6728–6732.

[8] J. Brzdęk and K. Ciepliński, *Hyperstability and superstability*, Abstr. Appl. Anal., **2013** (2013), Art. ID. 401756, 13 pp.

[9] K. Ciepliński, *On the generalized Hyers-Ulam stability of multi-quadratic mappings*, Comput. Math. Appl., **62** (2011), 3418–3426.

[10] S. Czerwik, *On the stability of the quadratic mapping in normed spaces*, Abh. Math. Sem. Hamburg, **62** (1992), 59–64.

[11] N. J. Daras and Th. M. Rassias, *Approximation and computation in science and Eengineering*, Series Springer Optimization and Its Applications (SOIA). Vol. **180**, Springer 2022.

[12] P. Găvruţa, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184**(3) (1994), 431–436.

[13] D. H. Hyers, *On the stability of the linear functional equation,* Proc. Nat. Acad. Sci. USA., **27** (1941), 222–224.

[14] P. Kannappan, *Functional equations and inequalities with applications*, Springer, 2009.

[15] C. Park, A. Bodaghi, and I. A. Alias, *Random stability and hyperstability of multi-quadratic mappings*, J. Math. Inequ., **16**(3) (2022), 993–1004.

[16] J. M. Rassias, E. Sathya, and M. Arunkumar,*Generalized Ulam-Hyers stability of an alternate additive-quadratic-quartic functional equation in fuzzy Banach spaces*, Math. Anal. Cont. Appl., **3**(1) (2021), 13–31.

[17] J. M. Rassias, M. Arunkumar, and E. Sathya, *Non-stabilities of mixed type Euler-Lagrange $k$-cubic-quartic functional equation in various normed spaces*, Math. Anal. Cont. Appl., **1**(1) (2019), 1–43.

[18] Th. M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72**(2) (1978), 297–300.

[19] P. K. Sahoo and P. Kannappan, *Introduction to functional equations*, CRC Press, Boca Raton, FL, 2011.

[20] F. Skof, *Proprieta locali e approssimazione di operatori* , Rend. Sem. Mat. Fis. Milano, **53** (1983), 113–129.

[21] S. Salimi and A. Bodaghi, *A fixed point application for the stability and hyperstability of multi-Jensen-quadratic mappings*, J. Fixed Point Theory Appl., **2020** (2020).

[22] S. M. Ulam, *Problems in modern mathematics,* Chapter VI, Science Ed., Wiley, New York, 1940.

[23] X. Zhao, X. Yang and C.-T. Pang, *Solution and stability of the multiquadratic functional equation*, Abstr. Appl. Anal., (2013) Art. ID 415053, 8 pp.

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