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Fixed point theorems in intuitionistic fuzzy 2-metric spaces

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ABSTRACT. In the paper, we find some fixed point theorem in intuitionistic fuzzy 2- metric spaces on four mappings and six mappings with the help of theory of sub compatible of type (A).

1. Introduction

Atanassov [3] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [15]. In 2004, Park [10] defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norms and continuous t-conorms. Recently, in 2006, Alaca et al. [1] Using the idea of intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous t-norm and continuous t-conorm as a generalization of fuzzy metric space due to Kramosil and Michalek [6]. Further, Alaca et al. [1] proved intuitionistic fuzzy Banach and intuitionistic fuzzy Edelstein contraction theorems, with the different definition of Cauchy sequences and completeness than the ones given . The idea of fuzzy 2-metric space and fuzzy 3-metric space was used by Sushil Sharma [13] and obtained some fruitful results. The concept of compatibility in fuzzy metric space is initiated by Singh and Chauhan and also derived some common fixed point theorem

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in that space. Jain and Singh [4], Jungck et al. [6] explained about fixed point theorem for compatible mappings of type (A) in fuzzy metric space. We find some fixed point theorem in intuitionistic fuzzy 2-metric spaces on four and six mappings with the help of sub compatible of type (A) with the help of example.

2. Preliminaries

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $*$ is satisfying the following conditions:
i) $*$ is a commutative and associative,
ii) $*$ is continuous,
iii) $a * 1 = a$ for all $a \in [0,1]$,
iv) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0,1]$.

Definition 2.2. A binary operation $\Diamond : [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t- conorm if it satisfies the following conditions:
(i) $\Diamond$ is associative and commutative.
(ii) $\Diamond$ is continuous.
(iii) $a\Diamond 0 = a$ for all $a \in [0,1]$
(iv) $a\Diamond b \leq c\Diamond d$ whenever $a \leq c$ and $b \leq d$, for each $a, b, c, d \in [0,1]$

Definition 2.3. A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,
(1) $M(x, y, t) + N(x, y, t) \leq 1$.
(2) $M(x, y, 0) = 0$ for all x, y in X.
(3) $M(x, y, t) = 1$ for all x, y in X and $t > 0$ if and only if $x = y$.
(4) $M(x, y, t) = M(y, x, t)$, for all x, y in X and $t > 0$.
(5) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$.
(6) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous.
(7) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y in X and $t > 0$.
(8) $N(x, y, 0) = 1$ for all x, y in X.
(9) $N(x, y, t) = 0$ for all x, y in X and $t > 0$ if and only if $x = y$.
(10) $N(x, y, t) = N(y, x, t)$, for all x, y in X and $t > 0$.
(11) $N(x, y, t)\Diamond N(y, z, s) \geq N(x, z, t + s)$.
(12) $N(x, y, .) : [0, \infty) \to [0, 1]$ is right continuous.
(13) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X and $t > 0$.
Then (M, N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and degree of non-nearness between x and y with respect to t, respectively.

Example 2.1. *Let (X, d) be a metric space. Define t-norm $a * b = \min\{a, b\}$ and t-conorm $a \Diamond b = \max\{a, b\}$ and for all $x, y \in X$ and $t > 0, M_d(x, y, t) = \frac{t}{t+d(x,y)}, N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$. Then $(X, M, N, *, \Diamond)$ is an IFM-space and the intuitionistic fuzzy metric space (M, N) induced by the metric d is often referred to as the standard intuitionistic fuzzy metric.*

Definition 2.4. A sequence $\{x_n\}$ in a intuitionistic fuzzy 2-metric space $(X, M, N, *, \Diamond)$ is said to be converge to x in X iff $\lim_{n\to\infty} M(x_n, x, a, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, a, t) = 0$, for all $a \in X$ and $t > 0$.

Definition 2.5. Let $(X, M, N, *, \Diamond)$ be a intuitionistic fuzzy 2- metric space. A sequence $\{x_n\}$ in X is called Cauchy sequence, iff

$$\lim_{n\to\infty} M(x_{n+p}, x_p, a, t) = 1 \text{ and } \lim_{n\to\infty} N(x_{n+p}, x_p, a, t) = 0,$$

for all $a \in X$ and $p > 0, t > 0$.

Definition 2.6. A intuitionistic fuzzy 2-metric space $(X, M, N, *, \Diamond)$ is said to be complete iff every Cauchy sequence in X is convergent in X.

Lemma 2.2. *In an intuitionistic fuzzy metric space $X, M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.*

Lemma 2.3. *Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. If there exists a constant $k \in (0, 1)$ such that*

$$M(x, y, a, kt) \geq M(x, y, a, t), N(x, y, a, kt) \leq N(x, y, a, t),$$

for $x, y \in X$. Then $x = y$.

Definition 2.7. Self-mappings A and S of a intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be sub compatible if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z, z \in X$ and satisfy $\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1$ and $\lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$.

3. Main Results

Theorem 3.1. *Consider four self-mappings A, B, U and V of a intuitionistic fuzzy 2- metric space $(X, F, G, *, \Diamond)$ with continuous t-norm $*$ and continuous t-corm $\Diamond$ defined by $t * t \geq t$ and $(1 - t)\Diamond(1 - t) \leq (1 - t)$ every one $t, 0 \leq t \leq 1$. If the couples (A, U) and (B, V) be sub compatible of type (A) with the identical coincidence points and $AB = BA, BU = UB, UV = VU, AU = UA, AV = VA$, for all $x, y, z \in X, k \in (0, 1)$ and $t > 0$ with*

(3.1.1) $F(Ux, Vy, z, kt) \geq \min \left\{ \begin{array}{l} F(Ax, By, z, t), F(Ax, Ux, z, t), \\ F(Vx, By, z, t), F(Ux, Ay, z, t) \end{array} \right\},$

(3.1.2) $G(Ux,Vy,z,kt) \leq \max\left\{\begin{array}{c} G(Ax,By,z,t), G(Ax,Ux,z,t), \\ G(Vx,By,z,t), G(Ux,Ay,z,t) \end{array}\right\}.$

Then in X, the maps A, B, U and V have common unique Fixed Point.

Proof. We know that the couples (A,U) and (B,V) be sub compatible of type (A) then readily available 2-sequences $\{x_n\}$ and $\{y_n\}$ inside X with

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Ux_n = a,$$

where a is in X and

$$\lim_{n\to\infty} F(AUx_n, UUx_n, z, t) = 1, \lim_{n\to\infty} G(AUx_n, UUx_n, z, t) = 0,$$

also

$$\lim_{n\to\infty} F(UAx_n, UUx_n, z, t) = 1, \lim_{n\to\infty} G(UAx_n, UUx_n, z, t) = 0.$$

Hence

$$\lim_{n\to\infty} F(Aa, Ua, z, t) = 1, \lim_{n\to\infty} G(Aa, Ua, z, t) = 0,$$

$$\lim_{n\to\infty} F(Ua, Aa, z, t) = 1 \text{ and } \lim_{n\to\infty} G(Ua, Aa, z, t) = 0$$

and

$$\lim By_n = \lim Vy_n = b,$$

b is in X and hold the following limit,

$$\lim_{n\to\infty} F(BVy_n, VVy_n, z, t) = 1, \lim_{n\to\infty} G(BVy_n, VVy_n, z, t) = 0,$$

$$\lim_{n\to\infty} F(VBy_n, VVy_n, z, t) = 1 \text{ and } \lim_{n\to\infty} G(VBy_n, VVy_n, z, t) = 0.$$

Hence $\lim_{n\to\infty} F(Bb, Vb, z, t) = 1$, $\lim_{n\to\infty} G(Bb, Vb, z, t) = 0$, $\lim_{n\to\infty} F(Vb, Bb, z, t) = 1$ and $\lim_{n\to\infty} G(Vb, Bb, z, t) = 0$.

Thus $Aa = Ua \,\&\, Bb = Vb$, here a and b respectively are coincidence points of A, U and B, V. At the present we are to demonstrate that $a = b$, we substitute $x = x_n$ and $y = y_n$ in (3.1.1) and (3.1.2) we get

$$F(Ux_n, Vy_n, z, kt) \geq \min\left\{\begin{array}{c} F(Ax_n, By_n, z, t), F(Ax_n, Ux_n, z, t), \\ F(Vy_n, By_n, z, t), F(Ux_n, Ay_n, z, t) \end{array}\right\},$$

$$G(Ux_n, Vy_n, z, kt) \leq \max\left\{\begin{array}{c} G(Ax_n, By_n, z, t), G(Ax_n, Ux_n, z, t), \\ G(Vy_n, By_n, z, t), G(Ux_n, Ay_n, z, t) \end{array}\right\}.$$

Applying the limiting as n tends to infinity, we obtain

$$F(a, b, z, kt) \geq \min\left\{\begin{array}{c} F(a, b, z, t), F(a, a, z, t), \\ F(b, b, z, t), F(a, b, z, t) \end{array}\right\},$$

$$G(a, b, z, kt) \leq \max\left\{\begin{array}{c} G(a, b, z, t), G(a, a, z, t), \\ G(b, b, z, t), G(a, b, z, t) \end{array}\right\}.$$

This shows that

$$F(a,b,z,kt) \geq F(a,b,z,t) \text{ and } G(a,b,z,kt) \leq G(a,b,z,t) \text{ for all } t > 0.$$

Hence besides using lemma (2.3), a equals to b. This indicates that the maps A, B, U and V have the identical coincidence point. After that we are to demonstrate $Aa = Ba = Ua = Va = a$.

First we take $x = a$ and also $y = y_n$ in the equation (3.1.1) and (3.1.2), we search out

$$F(Ua, Vy_n, z, kt) \geq \min \left\{ \begin{array}{c} F(Aa, By_n, z, t), F(Aa, Ua, z, t), \\ F(Vy_n, By_n, z, t), F(Ua, Ay_n, z, t) \end{array} \right\},$$

$$G(Ua, Vy_n, z, kt) \leq \max \left\{ \begin{array}{c} G(Aa, By_n, z, t), G(Aa, Ua, z, t), \\ G(Vy_n, By_n, z, t), G(Ua, Ay_n, z, t) \end{array} \right\}.$$

Applying limit as $n \to \infty$ on both side, we get

$$F(Ua, b, z, kt) \geq \min \left\{ \begin{array}{c} F(Aa, b, z, t), F(Aa, Ua, z, t), \\ F(b, b, z, t), F(Ua, b, z, t) \end{array} \right\},$$

$$\geq \min \left\{ \begin{array}{c} F(Ua, b, z, t), F(Ua, Ua, z, t), \\ F(b, b, z, t), F(Ua, b, z, t) \end{array} \right\}.$$

$$G(Ua, b, z, kt) \leq \max \left\{ \begin{array}{c} G(Aa, b, z, t), G(Aa, Ua, z, t), \\ G(b, b, z, t), G(Ua, b, z, t) \end{array} \right\},$$

$$\leq \max \left\{ \begin{array}{c} G(Ua, b, z, t), G(Ua, Ua, z, t), \\ G(b, b, z, t), G(Ua, b, z, t) \end{array} \right\}.$$

As $a = b$ then

$$F(Ua, a, z, kt) \geq F(Ua, a, z, t) \text{ and } G(Ua, a, z, kt) \leq G(Ua, a, z, t).$$

This gives, $Ua = a$ ie., $Ua = Aa = a$. Now, we substitute $x = x_n$ and $y = a$ in (3.1.1) and (3.1.2), we obtain

$$F(Ux_n, Va, z, kt) \geq \min \left\{ \begin{array}{c} F(Ax_n, Ba, z, t), F(Ax_n, Ux_n, z, t), \\ F(Va, Ba, z, t), F(Ux_n, Aa, z, t) \end{array} \right\},$$

$$G(Ux_n, Va, z, kt) \leq \max \left\{ \begin{array}{c} G(Ax_n, Ba, z, t), G(Ax_n, Ux_n, z, t), \\ G(Va, Ba, z, t), G(Ux_n, Aa, z, t) \end{array} \right\}.$$

Now, limiting on both side as $n \to \infty$, we obtain

$$F(a, Va, z, kt) \geq \min\left\{ \begin{array}{c} F(a, Ba, z, t), F(a, a, z, t), \\ F(Va, Ba, z, t), F(a, Aa, z, t \end{array} \right\},$$

$$\geq \min\left\{ \begin{array}{c} F(a, a, z, t), F(a, a, z, t), \\ F(Va, a, z, t), F(a, a, z, t) \end{array} \right\}.$$

$$\Rightarrow F(Va, a, z, kt) \geq F(Va, a, z, t).$$

$$G(a, Va, z, kt) \leq \max\left\{ \begin{array}{c} G(a, Ba, z, t), G(a, a, z, t), \\ G(Va, Ba, z, t), G(a, Aa, z, t) \end{array} \right\},$$

$$\leq \max\left\{ \begin{array}{c} G(a, a, z, t), G(a, a, z, t), \\ G(Va, a, z, t), G(a, a, z, t) \end{array} \right\}.$$

$$\Rightarrow G(Va, a, z, kt) \leq G(Va, a, z, t).$$

Which gives, $Va = a$, that is $Va = Ba = a$. Hence we get $Aa = Ua = Va = Ba = a$. □

Theorem 3.2. *Consider six self-mappings P, A, Q, B, P, T and S of a intuitionistic fuzzy 2- metric space $(X, F, G, *, \Diamond)$ with continuous t-norm $*$ and continuous t-corm $\Diamond$ defined by $t*t \geq t$ and $(1-t)\Diamond(1-t) \leq (1-t)$, for every one t in $[0, 1]$. If the couples $(AB, S)\&(PQ, T)$ be sub compatible of type(A) with equal coincidence points and $BS = SB, AB = BA, PQ = QP, TQ = QT, AS = SA, AT = TA, PT = TP$ for all x, y, z in X and for k in $(0, 1)$ and $t > 0$,*

(3.2.1) $F(Sx, Ty, z, kt) \geq \min\left\{ \begin{array}{c} F(ABx, PQy, z, t), F(ABx, Sx, z, t), \\ F(Tx, PQy, z, t), F(Sx, ABy, z, t) \end{array} \right\}$

(3.2.2) $G(Sx, Ty, z, kt) \leq \max\left\{ \begin{array}{c} G(ABx, PQy, z, t), G(ABx, Sx, z, t), \\ G(Tx, PQy, z, t), G(Sx, ABy, z, t) \end{array} \right\}$

at that moment the mappings A, B, P, Q, S and T have a common Fixed Point in X which is unique also.

Proof. We know that the couples $(AB, S)\,\&\,(PQ, T)$ be sub compatible of kind A then there exist 2 sequences $\{x_n\}$ and $\{y_n\}$ in X with the property $\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Sx_n = a, a \in X$ and $\lim_{n\to\infty} F(ABSx_n, SSx_n, z, t) = 1$;

$$\lim_{n\to\infty} G(ABSx_n, SSx_n, z, t) = 0; \lim_{n\to\infty} F(SABx_n, ABABx_n, z, t) = 1$$

$$\lim_{n\to\infty} G(SABx_n, ABABx_n, z, t) = 0.$$

Hence

$$\lim_{n\to\infty} F(ABa, Sa, z, t) = 1; \lim_{n\to\infty} G(ABa, Sa, z, t) = 0; \lim_{n\to\infty} F(Sa, ABa, z, t) = 1,$$

$$\lim_{n\to\infty} G(Sa, ABa, z, t) = 0; \lim_{n\to\infty} PQy_n = \lim_{n\to\infty} Ty_n = b, b \in X$$

and satisfy

$$\lim_{n\to\infty} F(PQTy_n, TTy_n, z, t) = 1; \lim_{n\to\infty} G(PQTy_n, TTy_n, z, t) = 0$$

$$\lim_{n\to\infty} F(TPQy_n, PQPQy_n, z, t) = 1, \lim_{n\to\infty} G(TPQy_n, PQPQy_n, z, t) = 0.$$

Consequently, we obtain

$$\lim_{n\to\infty} F(PQb, Tb, z, t) = 1, \lim_{n\to\infty} G(PQb, Tb, z, t) = 0, \lim_{n\to\infty} F(Tb, PQb, z, t) = 1$$

and

$$\lim_{n\to\infty} G(Tb, PQb, z, t) = 0.$$

So, $ABa = Sa$ and $PQb = Tb$. Thus coincidence point of AB and S is "a" and coincidence point of PQ and T is "b".

At the present, we have to show that $a = b$, for this substitute $x = x_n$ and $y = y_n$ in (3.2.1) and (3.2.2), we obtain

$$F(Sx_n, Ty_n, z, kt) \geq \min \left\{ \begin{array}{c} F(ABx_n, PQy_n, z, t), F(ABx_n, Sx_n, z, t), \\ F(Ty_n, PQy_n, z, t), F(Sx_n, ABy_n, z, t) \end{array} \right\},$$

$$G(Sx_n, Ty_n, z, kt) \leq \max \left\{ \begin{array}{c} G(ABx_n, PQy_n, z, t), G(ABx_n, Sx_n, z, t), \\ G(Ty_n, PQy_n, z, t), G(Sx_n, ABy_n, z, t) \end{array} \right\}.$$

Considering $n \to \infty$ as limiting value, we come across that

$$F(a, b, z, kt) \geq \min \left\{ \begin{array}{c} F(a, b, z, t), F(a, a, z, t), \\ F(b, b, z, t), F(a, b, z, t) \end{array} \right\},$$

$$G(a, b, z, kt) \leq \max \left\{ \begin{array}{c} G(a, b, z, t), G(a, a, z, t), \\ G(b, b, z, t), G(a, b, z, t) \end{array} \right\}.$$

This shows that

$$F(a, b, z, kt) \geq F(a, b, z, t) \text{ and } G(a, b, z, kt) \leq G(a, b, z, t), \text{ for all } t > 0.$$

Then by means of lemma (2.3), a is equal to b. This implies that AB, S, PQ and T have the equal coincidence point. Next we have to prove that $Aa = Ba = Pa = Qa = Sa = Ta = a$. First, we substitute $x = a$ and $y = y_n$ in (3.2.1)and (3.2.2), we obtain

$$F(Sa, Ty_n, z, kt) \geq \min \left\{ \begin{array}{c} F(ABa, PQy_n, z, t), F(ABa, Sa, z, t), \\ F(Ty_n, PQy_n, z, t), F(Sa, ABy_n, z, t) \end{array} \right\},$$

$$G(Sa, Ty_n, z, kt) \leq \max \left\{ \begin{array}{c} G(ABa, PQy_n, z, t), G(ABa, Sa, z, t), \\ G(Ty_n, PQy_n, z, t), G(Sa, ABy_n, z, t) \end{array} \right\}.$$

Applying limit as $n \to \infty$, we obtain

$$F(Sa, b, z, kt) \geq \min \left\{ \begin{array}{c} F(ABa, b, z, t), F(ABa, Sa, z, t), \\ F(b, b, z, t), F(Sa, b, z, t) \end{array} \right\},$$

$$G(Sa, b, z, kt) \leq \max \left\{ \begin{array}{c} G(ABa, b, z, t), G(ABa, Sa, z, t), \\ G(b, b, z, t), G(Sa, b, z, t) \end{array} \right\}.$$

As $a = b$,

$$F(Sa, a, z, kt) \geq F(Sa, a, z, t) \, \text{and} \, G(Sa, a, z, kt) \leq G(Sa, a, z, t)$$

which shows that $Sa = a$. Now, consider $x = x_n$ and $y = a$ in (3.2.1) and (3.2.2), we obtain

$$F(Sx_n, Ta, z, kt) \geq \min \left\{ \begin{array}{c} F(ABx_n, PQa, z, t), F(ABx_n, Sx_n, z, t), \\ F(Ta, PQa, z, t), F(Sx_n, ABa, z, t) \end{array} \right\},$$

$$G(Sx_n, Ta, z, kt) \leq \max \left\{ \begin{array}{c} G(ABx_n, PQa, z, t), G(ABx_n, Sx_n, z, t), \\ G(Ta, PQa, z, t), G(Sx_n, ABa, z, t) \end{array} \right\}.$$

Now, apply limit as $n \to \infty$, we obtain

$$\begin{aligned} F(a, Ta, z, kt) &\geq \min \left\{ \begin{array}{c} F(a, PQa, z, t), F(a, a, z, t), \\ F(Ta, PQa, z, t), F(a, ABa, z, t) \end{array} \right\} \\ &\geq \min \left\{ \begin{array}{c} F(a, Ta, z, t), F(a, a, z, t), \\ F(Ta, Ta, z, t), F(a, Sa, z, t) \end{array} \right\} \\ &\geq F(Ta, a, z, t), \\ G(a, Ta, z, kt) &\leq \max \left\{ \begin{array}{c} G(a, PQa, z, t), G(a, a, z, t), \\ G(Ta, PQa, z, t), G(a, ABa, z, t) \end{array} \right\} \\ &\leq \max \left\{ \begin{array}{c} G(a, Ta, z, t), G(a, a, z, t), \\ G(Ta, Ta, z, t), G(a, Sa, z, t) \end{array} \right\} \\ &\leq G(Ta, a, z, t). \end{aligned}$$

This implies that $Ta = a$. We show that $Aa = Ba = a$. Putting $x = Ba$ and $y = y_n$ in (3.2.1) and (3.2.2), we obtain

$$F(SBa, Ty_n, z, kt) \geq \min \left\{ \begin{array}{c} F(ABBa, PQy_n, z, t), F(ABBa, SBa, z, t), \\ F(Ty_n, PQy_n, z, t), F(SBa, ABy_n, z, t) \end{array} \right\},$$

$$G(SBa, Ty_n, z, kt) \leq \max \left\{ \begin{array}{c} G(ABBa, PQy_n, z, t), G(ABBa, SBa, z, t), \\ G(Ty_n, PQy_n, z, t), G(SBa, ABy_n, z, t) \end{array} \right\}.$$

We know that A, B and S commutes each other. So, $ABBa = BABa = BSa = Ba$.

$$F(Ba, a, z, kt) \geq \min \left\{ \begin{array}{c} F(Ba, a, z, t), F(Ba, Ba, z, t), \\ F(a, a, z, t), F(Ba, a, z, t) \end{array} \right\},$$
$$F(Ba, a, z, kt) \geq F(Ba, a, z, t),$$
$$G(Ba, a, z, kt) \leq \max \left\{ \begin{array}{c} G(Ba, a, z, t), G(Ba, Ba, z, t), \\ G(a, a, z, t), G(Ba, a, z, t) \end{array} \right\},$$
$$G(Ba, a, z, kt) \leq G(Ba, a, z, t).$$

Now put $x = Aa$ and $y = y_n$ in (3.2.1) and (3.2.2), we get

$$F(SAa, Ty_n, z, kt) \geq \min \left\{ \begin{array}{c} F(ABAa, PQy_n, z, t), F(ABAa, SAa, z, t), \\ F(Ty_n, PQy_n, z, t), F(SAa, ABy_n, z, t) \end{array} \right\},$$
$$G(SAa, Ty_n, z, kt) \leq \max \left\{ \begin{array}{c} G(ABAa, PQy_n, z, t), G(ABAa, SAa, z, t), \\ G(Ty_n, PQy_n, z, t), G(SAa, ABy_n, z, t) \end{array} \right\}.$$

As A, B and S commutes $SAa = ASa = Aa$ and $ABAa = ASa = Aa$.

$$F(Aa, PQy_n, z, kt) \geq \min \left\{ \begin{array}{c} F(Aa, PQy_n, z, t), F(Aa, Aa, z, t), \\ F(Ty_n, PQy_n, z, t), F(Aa, ABy_n, z, t) \end{array} \right\},$$
$$F(Aa, a, z, kt) \geq \min \left\{ \begin{array}{c} F(Aa, a, z, t), F(Aa, Aa, z, t), \\ F(a, a, z, t), F(Aa, a, z, t) \end{array} \right\},$$
$$F(Aa, a, z, kt) \geq F(Aa, a, z, t),$$
$$G(Aa, PQy_n, z, kt) \leq \max \left\{ \begin{array}{c} G(Aa, PQy_n, z, t), G(Aa, Aa, z, t), \\ G(Ty_n, PQy_n, z, t), G(Aa, ABy_n, z, t) \end{array} \right\},$$
$$G(Aa, a, z, kt) \leq \max \left\{ \begin{array}{c} G(Aa, a, z, t), G(Aa, Aa, z, t), \\ G(a, a, z, t), G(Aa, a, z, t) \end{array} \right\},$$
$$G(Aa, a, z, kt) \leq G(Aa, a, z, t).$$

Therefore, Aa = a. Hence we have $Aa = Ba = Sa = a$. Similarly to show that $Qa = a$, we substitute $x = x_n$ and $y = Qa$ and to show that $Pa = a$, put $x = x_n$ and $y = Pa$. Hence we obtain $Pa = Aa = Ba = Ta = Qa = Sa = a$. □

Example 3.3. *Let self-mappings of X be P, Q, A, B, T and S and let $X = [0, 1]$, where $Ax = \frac{x}{3}, Bx = \frac{x}{2}, Sx = \frac{x}{6}, Tx = \frac{x}{6}, Qx = 2x$ and $Px = \frac{x}{12}$. Let $\{x_n\}$ and $\{y_n\}$ be 2 sequences, where $x_n = \frac{n}{n+1}, y_n = \frac{n^2}{n^2+1}$. Then $\frac{1}{6}$ is the fixed point of P, Q, A, B, T and S.*

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