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Notes on the stability of multimixed additive-quartic mappings

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Abstract. In this article, we prove the Hyers–Ulam stability of multimixed additive-quartic functional equations in the setting of Banach spaces by applying a fixed point method and moreover we generalize some known results.

1. Introduction

In 1940, Ulam [25] gave a wide-ranging discussion in which he explored a variety of major unresolved problems. One of these was the subject of the stability of homomorphisms. Recall that an equation is *stable* in some class of functions if any function from that class, satisfying the equation approximately (in some sense), is near (in some way) to an exact solution of the equation. In 1941, Hyers [16] solved the Ulam problem for Banach spaces for the first time. Then, Th. M. Rassias [24] was able to expand Hyers' conclusion. A generalization of the Rassias theorem was obtained by Găvruţa [15] by replacing the unbounded Cauchy difference by a general control function in the spirit of Rassias approach. The concept Hyers–Ulam stability derives from these historical contexts; see [22] and [23] and references therein.

Let V be a commutative group, W be a linear space over rational numbers, and n be an integer with $n \geq 2$. A mapping $f : V^n \longrightarrow W$ is called *multiadditive* if it satisfies the Cauchy's functional equation $A(x + y) = A(x) + A(y)$ in each variable. More information about the structure of multi-additive mappings are available for instance in [12] and [17]. Moreover, f is said to be *multiquartic* if it satisfies one of

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the equations

$$\mathfrak{Q}(x+2y)+\mathfrak{Q}(x-2y)=4\mathfrak{Q}(x+y)+4\mathfrak{Q}(x-y)-6\mathfrak{Q}(x)+24\mathfrak{Q}(y) \tag{1}$$

or

$$\mathfrak{Q}(2x+y)+\mathfrak{Q}(2x-y)=4\mathfrak{Q}(x+y)+4\mathfrak{Q}(x-y)+24\mathfrak{Q}(x)-6\mathfrak{Q}(y), \tag{2}$$

in each variable. For more details about quartic, multiquartic functional equation and their stabilities see [1, 7, 18, 19, 21].

In the last two decades, the stability problem for several variables mappings such as multiadditive, multiquadratic, multicubic and multiquartic mappings and functional equations by applying direct and fixed point methods have been studied by a number of authors which are available for example in [2], [3], [8], [10], [11], [13], [20], and [26].

In [14], Eshaghi Gordji introduced and obtained the general solution of the following mixed type additive and quartic functional equation

$$\begin{aligned} &f(2x+y)+f(2x-y)\\ &\quad =4[(f(x+y)+f(x-y)]-\frac{3}{7}(f(2y)-2f(y))+2f(2x)-8f(x). \end{aligned} \tag{3}$$

A alternative form of mixed type additive and quartic functional equation has been introduced by Bodaghi in [5] as follows:

$$f(x+2y)-4f(x+y)-4f(x-y)+f(x-2y)=\frac{12}{7}(f(2y)-2f(y))-6f(x). \tag{4}$$

It is easily verified that the function $f(x)=\alpha x^4+\beta x$ is a solution of the equations (3) and (4); the generalized version of equation (4) can be found in [4]. Recently, motivated by equation (4), Bodaghi et al. [6] defined the multimixed additive-quartic mappings and characterized the general form of such mappings as a equation. In other words, they unified the system of n equations defining the multi-mixed additive-quartic mappings to a single equation.

In this paper, we prove the Hyers–Ulam stability for the multi-mixed additive-quartic mappings in the setting of Banach spaces by applying a fixed point method [9]. In other word, we generalize some results in [6]. As a consequence, we show that every multimixed additive-quartic mapping is δ-stable for a small positive number δ.

2. Main Results

Throughout this section, $\mathbb{N}$ stands for the set of all positive integers, $\mathbb{N}_0 := \mathbb{N}\cup\{0\}, \mathbb{R}_+ := [0,\infty), n\in\mathbb{N}$. Moreover, V and W are vector spaces over the rational numbers$\mathbb{Q}$, $n\in\mathbb{N}$ and $x_i^n=(x_{i1},x_{i2},\ldots,x_{in})\in V^n$, where $i\in\{1,2\}$. We shall denote x_i^n by x_i or simply x if there is no risk of ambiguity.

Definition 2.1. [6] A mapping $f : V^n \longrightarrow W$ is called *n-multimixed additive-quartic* or briefly *multimixed additive-quartic* if f satisfies mixed additive-quartic equation (4) in each variable.

Let $n \in \mathbb{N}$ with $n \geq 2$ and $x_i^n = (x_{i1}, x_{i2}, \dots, x_{in}) \in V^n$, where $i \in \{1, 2\}$. For $x_1, x_2 \in V^n$ and $p_i, q \in \mathbb{N}_0$ with $0 \leq p_i, q \leq n$, put

$$\mathcal{M} = \{\mathfrak{M}_n = (M_1, \dots, M_n) |\ M_j \in \{x_{1j} \pm 2x_{2j}, 2x_{2j}\},\ j \in \{1, \dots, n\}\},$$

and

$$\mathcal{N} = \{\mathfrak{N}_n = (N_1, \dots, N_n) |\ N_j \in \{x_{1j} \pm x_{2j}, x_{1j}, x_{2j}\}\}.$$

Consider the subsets $\mathcal{M}_q^n$ and $\mathcal{N}_{(p_1,p_2)}^n$ of $\mathcal{M}$ and $\mathcal{N}$, respectively, as follows:

$$\mathcal{M}_q^n := \{\mathfrak{M}_n \in \mathcal{M} |\ \mathrm{Card}\{M_j :\ M_j = 2x_{2j}\} = q\}.$$

$$\mathcal{N}_{(p_1,p_2)}^n := \{\mathfrak{N}_n \in \mathcal{N} |\ \mathrm{Card}\{N_j :\ N_j = x_{ij}\} = p_i\ \ (i \in \{1, 2\})\},$$

where $j \in \{1, \dots, n\}$. From now on, for the multimixed additive-quartic mappings, we use the following notations:

$$f\left(\mathcal{M}_q^n\right) := \sum_{\mathfrak{M}_n \in \mathcal{M}_q^n} f(\mathfrak{M}_n). \tag{5}$$

$$f\left(\mathcal{N}_{(p_1,p_2)}^n\right) := \sum_{\mathfrak{N}_n \in \mathcal{N}_{(p_1,p_2)}^n} f(\mathfrak{N}_n). \tag{6}$$

The following result was proved in [6].

Proposition 2.1. *If a mapping $f : V^n \longrightarrow W$ is multimixed additive-quartic, it satisfies the equation*

$$\sum_{q=0}^{n} \left(-\frac{12}{7}\right)^q f\left(\mathcal{M}_q^n\right) = \sum_{p_1=0}^{n} \sum_{p_2=0}^{n-p_1} 4^{n-p_1-p_2}(-6)^{p_1} \left(-\frac{24}{7}\right)^{p_2} f\left(\mathcal{N}_{(p_1,p_2)}^n\right), \tag{7}$$

where $f\left(\mathcal{M}_q^n\right)$ and $f\left(\mathcal{N}_{(p_1,p_2)}^n\right)$ are defined in (5) and (6), respectively.

In continuation, we prove some Hyers–Ulam stability results by a fixed point method in the setting of Banach spaces. In what follows, we denote the set of all mappings from E to F by F^E. We remember the following theorem which is an essential result in fixed point theory [10, Theorem 1]. This achievement is a key tool in obtaining our aim in this section.

Theorem 2.2. *Let the hypotheses*

(H1) *Y is a Banach space, E is a nonempty set, $j \in \mathbb{N}$, $g_1, \dots, g_j : E \longrightarrow E$ and $L_1, \dots, L_j : E \longrightarrow \mathbb{R}_+$,* (H2) *$\mathcal{T}: Y^E \longrightarrow Y^E$ is an operator satisfying the inequality*

$$\|\mathcal{T}\lambda(x) - \mathcal{T}\mu(x)\| \leq \sum_{i=1}^{j} L_i(x) \|\lambda(g_i(x)) - \mu(g_i(x))\|, \quad \lambda, \mu \in Y^E, x \in E,$$

(H3) *$\Lambda : \mathbb{R}_+^E \longrightarrow \mathbb{R}_+^E$ is an operator defined through*

$$\Lambda\delta(x) := \sum_{i=1}^{j} L_i(x)\delta(g_i(x)) \qquad \delta \in \mathbb{R}_+^E, x \in E.$$

hold and a function $\theta : E \longrightarrow \mathbb{R}_+$ and a mapping $\phi : E \longrightarrow Y$ fulfill the following two conditions:

$$\|\mathcal{T}\phi(x) - \phi(x)\| \leq \theta(x), \quad \theta^*(x) := \sum_{l=0}^{\infty} \Lambda^l \theta(x) < \infty \qquad (x \in E).$$

Then, there exists a unique fixed point ψ of $\mathcal{T}$ such that

$$\|\phi(x) - \psi(x)\| \leq \theta^*(x) \qquad (x \in E).$$

Moreover, $\psi(x) = \lim_{l\to\infty} \mathcal{T}^l \phi(x)$ for all $x \in E$.

We say a mapping $f : V^n \longrightarrow W$

(i) has *zero condition* if $f(x) = 0$ for any $x \in V^n$ with at least one component which is equal to zero.

(ii) is odd in the jth variable if

$$f(z_1, \dots, z_{j-1}, -z_j, z_{j+1}, \dots, z_n) = -f(z_1, \dots, z_{j-1}, z_j, z_{j+1}, \dots, z_n).$$

(iii) is even in the jth variable if

$$f(z_1, \dots, z_{j-1}, -z_j, z_{j+1}, \dots, z_n) = f(z_1, \dots, z_{j-1}, z_j, z_{j+1}, \dots, z_n).$$

For the rest of this paper and for each mapping $f : V^n \longrightarrow W$, we consider the difference operator $\mathcal{D}_{AQ} f : V^n \times V^n \longrightarrow W$ defined via

$$\mathcal{D}_{AQ} f(x_1, x_2) := \sum_{q=0}^{n} \left(-\frac{12}{7}\right)^q f\left(\mathcal{M}_q^n\right) - \sum_{p_1=0}^{n} \sum_{p_2=0}^{n-p_1} 4^{n-p_1-p_2}(-6)^{p_1} \left(-\frac{24}{7}\right)^{p_2} f\left(\mathcal{N}_{(p_1,p_2)}^n\right),$$

where $f\left(\mathcal{M}_q^n\right)$ and $f\left(\mathcal{N}_{(p_1,p_2)}^n\right)$ are defined in (5) and (6), respectively. In the sequel, all mappings $f : V^n \longrightarrow W$ are assumed that satisfy zero condition. With this assumption, we have the next stability result for functional equation (7).

Theorem 2.3. *Let $\beta \in \{-1, 1\}$ be fixed, V be a linear space and W be a Banach space. Suppose that $\phi : V^n \times V^n \longrightarrow \mathbb{R}_+$ is a mapping satisfying*

$$\lim_{l\to\infty} \left(\frac{1}{2^{(4n-3k)\beta}}\right)^l \phi(2^{\beta l} x_1, 2^{\beta l} x_2) = 0, \tag{8}$$

for all $x_1, x_2 \in V^n$ *and*

$$\Phi(x) =: \frac{7^n}{2^{5n-2k} \times 3^k} \times \frac{1}{2^{(4n-3k)\frac{\beta-1}{2}}} \sum_{l=0}^{\infty} \left(\frac{1}{2^{(4n-3k)\beta}}\right)^l \phi\left(0, 2^{\beta l + \frac{\beta-1}{2}} x\right) < \infty, \tag{9}$$

for all $x \in V^n$. *Assume also* $f : V^n \longrightarrow W$ *is an odd mapping in each of some* k *variables and is even in each of the other variables and moreover satisfying the inequality*

$$\|\mathcal{D}_{AQ} f(x_1, x_2)\| \leqslant \phi(x_1, x_2), \tag{10}$$

for all $x_1, x_2 \in V^n$. *Then, there exists a unique solution* $\mathcal{F} : V^n \longrightarrow W$ *of* (7) *such that*

$$\|f(x) - \mathcal{F}(x)\| \leq \Phi(x), \tag{11}$$

for all $x \in V^n$.

Proof. Without loss of generality, we assume that f is odd in the k first of variables. Replacing (x_1, x_2) by $(0, x_1)$ in (10) and applying the hypotheses, we obtain

$$\left\| \left(-\frac{12}{7}\right)^k Sf(2x) - \left(-\frac{24}{7}\right)^k Tf(x) \right\| \leq \phi(0, x), \tag{12}$$

for all $x = x_1 \in V^n$ in which

$$S = \sum_{q=0}^{n-k} \binom{n-k}{q} \left(-\frac{12}{7}\right)^q 2^{n-k-q} \tag{13}$$

and

$$T = \sum_{p=0}^{n-k} \binom{n-k}{p} \left(-\frac{24}{7}\right)^p 4^{n-k-p} \times 2^{n-k-p}. \tag{14}$$

We have

$$S = \sum_{q=0}^{n-k} \binom{n-k}{q} \left(-\frac{12}{7}\right)^q 2^{n-k-q} = \left(-\frac{12}{7} + 2\right)^k = \left(\frac{2}{7}\right)^k$$

and

$$T = \sum_{p=0}^{n-k} \binom{n-k}{p} \left(-\frac{24}{7}\right)^p 4^{n-k-p} \times 2^{n-k-p} = \left(-\frac{24}{7} + 8\right)^{n-k} = \left(\frac{32}{7}\right)^{n-k}.$$

By the relations above, (12) will be

$$\left\| \left(-\frac{12}{7}\right)^k \left(\frac{2}{7}\right)^{n-k} f(2x) - \left(-\frac{24}{7}\right)^k \left(\frac{32}{7}\right)^{n-k} f(x) \right\| \leq \phi(0, x),$$

and so

$$\left\|f(x)-\frac{1}{2^{4n-3k}}f(2x)\right\| \leq \frac{7^n}{3^k\times 2^{5n-2k}}\phi(0,x). \tag{15}$$

Set

$$\theta(x) := \frac{7^n}{2^{5n-2k}\times 3^k}\times\frac{1}{2^{(4n-3k)\frac{\beta-1}{2}}}\phi\left(0,2^{\frac{\beta-1}{2}}x\right),$$

and

$$\mathcal{T}\theta(x) := \frac{1}{2^{(4n-3k)\beta}}\theta(2^\beta x),$$

where $\theta\in W^{V^n}$. Then, relation (15) can be modified as

$$\|f(x)-\mathcal{T}f(x)\|\leq\theta(x) \qquad (x\in V^n).$$

Define $\Lambda\eta(x) := \frac{1}{2^{(4n-3k)\beta}}\eta(2^\beta x)$ for all $\eta\in\mathbb{R}_+^{V^n}$. It is seen that Λ has the form (H3) of Theorem 2.2 for which $E=V^n$, $g_1(x)=2^\beta x$ and $L_1(x)=\frac{1}{2^{(4n-3k)\beta}}$. Furthermore, for each $\lambda,\mu\in W^{V^n}$, we get

$$\|\mathcal{T}\lambda(x)-\mathcal{T}\mu(x)\| = \left\|\frac{1}{2^{(4n-3k)\beta}}\left[\lambda(2^\beta x)-\mu(2^\beta x)\right]\right\| \leq L_1(x)\left\|\lambda(g_1(x))-\mu(g_1(x))\right\|.$$

The last relation shows that the hypothesis (H2) holds. By induction on l, one can check that for any $l\in\mathbb{N}_0$, we have

$$\begin{aligned}\Lambda^l\theta(x) :&= \left(\frac{1}{2^{(4n-3k)\beta}}\right)^l\theta(2^{\beta l}x)\\ &= \frac{7^n}{2^{5n-2k}\times 3^k}\times\frac{1}{2^{(4n-3k)\frac{\beta-1}{2}}}\left(\frac{1}{2^{(4n-3k)\beta}}\right)^l\phi\left(0,2^{\beta l+\frac{\beta-1}{2}}x\right).\end{aligned} \tag{16}$$

It follows from (9) and (16) that all assumptions of Theorem 2.2 are satisfied and so there exists a unique mapping $\mathcal{F}:V^n\longrightarrow W$ such that

$$\mathcal{F}(x)=\lim_{l\to\infty}(\mathcal{T}^l f)(x)=\frac{1}{2^{(4n-3k)\beta}}\mathcal{F}(2^\beta x) \qquad (x\in V^n),$$

and moreover (11) is valid. Next, we show that

$$\|\mathcal{D}_{AQ}(\mathcal{T}^l f)(x_1,x_2)\| \leq \left(\frac{1}{2^{(4n-3k)\beta}}\right)^l\phi(2^{\beta l}x_1,2^{\beta l}x_2), \tag{17}$$

for all $x_1, x_2 \in V^n$ and $l \in \mathbb{N}_0$. We argue by induction on l. It is obvious that(17) is true for $l = 0$ by (10). Assume that (17) holds for an $l \in \mathbb{N}_0$. Then

$$\begin{aligned}
&\|\mathcal{D}_{AQ}(\mathcal{T}^{l+1} f)(x_1, x_2)\| \\
&= \Big\| \sum_{q=0}^{n} \left(-\frac{12}{7}\right)^q (\mathcal{T}^{l+1} f) \left(\mathcal{M}_q^n\right) - \sum_{p_1=0}^{n} \sum_{p_2=0}^{n-p_1} 4^{n-p_1-p_2} (-6)^{p_1} \left(-\frac{24}{7}\right)^{p_2} (\mathcal{T}^{l+1} f) \left(\mathcal{N}_{(p_1,p_2)}^n\right) \Big\| \\
&= \frac{1}{2^{(4n-3k)\beta}} \Big\| \sum_{q=0}^{n} \left(-\frac{12}{7}\right)^q (\mathcal{T}^{l} f) \left(2^\beta \mathcal{M}_q^n\right) \\
&\qquad\qquad - \sum_{p_1=0}^{n} \sum_{p_2=0}^{n-p_1} 4^{n-p_1-p_2} (-6)^{p_1} \left(-\frac{24}{7}\right)^{p_2} (\mathcal{T}^{l} f) \left(2^\beta \mathcal{N}_{(p_1,p_2)}^n\right) \Big\| \\
&= \frac{1}{2^{(4n-3k)\beta}} \left\|\mathcal{D}_{AQ}(\mathcal{T}^{l} f)(2^\beta x_1, 2^\beta x_2)\right\| \le \left(\frac{1}{2^{(4n-3k)\beta}}\right)^{l+1} \phi(2^{\beta(l+1)} x_1, 2^{\beta(l+1)} x_2),
\end{aligned}$$

for all $x_1, x_2 \in V^n$. Letting $l \to \infty$ in (17) and applying (8), we arrive at $\mathcal{D}_{AQ}\mathcal{F}(x_1, x_2) = 0$ for all $x_1, x_2 \in V^n$. This means that the mapping $\mathcal{F}$ satisfies (7). Finally, assume that $\mathfrak{F} : V^n \longrightarrow W$ is another mapping satisfying equation (7) and inequality (11), and fix $x \in V^n$, $j \in \mathbb{N}$. Then, by [6, Lemma 8] and (9), we have

$$\begin{aligned}
&\|\mathcal{F}(x) - \mathfrak{F}(x)\| \\
&= \left\| \left(\frac{1}{2^{4n-3k}}\right)^j \mathcal{F}(2^j x) - \left(\frac{1}{2^{4n-3k}}\right)^j \mathfrak{F}(2^j x) \right\| \\
&\le \left(\frac{1}{2^{4n-3k}}\right)^j \left(\|\mathcal{F}(2^j x) - f(2^j x)\| + \|\mathfrak{F}(2^j x) - f(2^j x)\|\right) \\
&\le 2 \left(\frac{1}{2^{4n-3k}}\right)^j \Phi(2^j x) \\
&\le 2 \frac{7^n}{2^{5n-2k} \times 3^k} \times \frac{1}{2^{(4n-3k)\frac{\beta-1}{2}}} \sum_{l=j}^{\infty} \left(\frac{1}{2^{(4n-3k)\beta}}\right)^l \phi\left(0, 2^{\beta l + \frac{\beta-1}{2}} x\right).
\end{aligned}$$

Consequently, letting $j \to \infty$ and using the fact that series (9) is convergent for all $x \in V^n$, we obtain $\mathcal{F}(x) = \mathfrak{F}(x)$ for all $x \in V^n$. This completes the proof. □

Putting $k = 0, n$ in Theorem 2.3, we reach to Theorems 13 and 14 from [6], and thus this theorem generalizes main theorems of [6].

Here and subsequently, it is assumed that V is a normed space and W is a Banach space. In the following corollary, we show that the multimixed additive-quartic mappings are stable if (7) is controlled by the powers of norms of variables.

Corollary 2.4. *Given $\alpha \in \mathbb{R}$ with $\alpha \neq 4n - 3k$. Suppose that $f : V^n \longrightarrow W$ is an odd mapping in each of some k variables and is even in each of the other variables* *and moreover satisfying the inequality*

$$\|\mathcal{D}_{AQ}f(x_1,x_2)\| \leq \sum_{i=1}^{2}\sum_{j=1}^{n}\|x_{ij}\|^{\alpha},$$

for all $x_1, x_2 \in V^n$. Then, there exists a unique solution $\mathcal{F}: V^n \longrightarrow W$ of (7) such that

$$\|f(x)-\mathcal{F}(x)\| \leq \frac{7^n}{2^{n+k}\times 3^k|2^{4n-3k}-2^{\alpha}|}\sum_{j=1}^{n}\|x_{1j}\|^{\alpha}$$

for all $x = x_1 \in V^n$.

The next corollary is a direct consequence of Theorems 2.3 when the functional equation (7) is controlled by a small positive number δ.

Corollary 2.5. *Let $\delta > 0$. Suppose that $f : V^n \longrightarrow W$ is an odd mapping in each of some k variables and is even in each of the other variables and moreover satisfying the inequality*

$$\|\mathcal{D}_{AQ}f(x_1,x_2)\| \leq \delta,$$

for all $x_1, x_2 \in V^n$. Then, there exists a unique solution $\mathcal{F}: V^n \longrightarrow W$ of (7) such that

$$\|f(x)-\mathcal{F}(x)\| \leq \frac{7^n}{2^{n+k}\times 3^k(2^{4n-3k}-1)}\delta$$

for all $x = x_1 \in V^n$.

Proof. Letting the constant function $\phi(x_1, x_2) = \delta$ for all $x_1, x_2 \in V^n$, and using Theorem 2.3 in the case $\beta = 1$, one can obtain the desired result. □

Note that Corollaries 15 and 16 of [6] are some special case of Corollary 2.5 in the cases $= 0$ and $k = n$.

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References

[1] Z. Abbasbeygi, A. Bodaghi and A. Gharibkhajeh, *On an equation characterizing multi-quartic mappings and its stability*, Int. J. Nonlinear Anal. Appl., **13**(1)(2022), 991-1002.

[2] A. Bahyrycz, K. Ciepliński and J. Olko, *On an equation characterizing multi-additive-quadratic mappings and its Hyers-Ulam stability.* Appl. Math. Comput., **265**(2015), 448-455.

[3] A. Bodaghi, *Functional inequalities for generalized multi-quadratic mappings*, J. Inequal. Appl., **2021**(2021), 145.

[4] A. Bodaghi, *Approximate mixed type additive and quartic functional equation*, Bol. Soc. Paran. Mat., **35**(1)(2017), 43-56.

[5] A. Bodaghi, *Stability of a mixed type additive and quartic function equation*, Filomat, **28**(8)(2014), 1629-1640.

[6] A. Bodaghi, I. A. Alias, L. Mousavi and S. Hosseini, *Characterization and stability of multimixed additive-quartic mappings: A fixed point application*, J. Funct. Spaces, **2021**(2021), Art. ID 9943199.

[7] A. Bodaghi, C. Park and O. T. Mewomo, *Multiquartic functional equations*, Adv. Difference Eq. **2019**(2019), 312.

[8] A. Bodaghi and B. Shojaee, *On an equation characterizing multi-cubic mappings and its stability and hyperstability*, Fixed Point Theory, **22**(1)(2021), 83-92.

[9] J. Brzdęk, *Stability of the equation of the p-Wright affine functions*, Aequat. Math., **85**(2013), 497-503.

[10] J. Brzdęk and J. Chudziak and Zs. Palés, *A fixed point approach to the stability of functional equations in non-Archimedean metric spaces*, Nonlinear Anal., **74**(2011), 6728-6732.

[11] K. Ciepliński, *On the generalized Hyers-Ulam stability of multi-quadratic mappings*, Comput. Math. Appl., **62**(2011), 3418-3426.

[12] K. Ciepliński, *Generalized stability of multi-additive mappings*, Appl. Math. Lett., **23**(2010), 1291-1294.

[13] N. Ebrahimi Hoseinzadeh, A. Bodaghi and M. R. Mardanbeigi, *Almost multi-cubic mappings and a fixed point application*, Sahand Commun. Math. Anal., **17**(3)(2020), 131-143.

[14] M. Eshaghi Gordji, *Stability of a functional equation deriving from quartic and additive functions*, Bull. Korean Math. Soc., **47**(3)(2010), 491-502.

[15] P. Găvruţa, *A generalization of the Hyers-Ulam-Rassias stability of approximately additive mappings*, J. Math. Anal. Appl., **184**(1994), 431-436.

[16] D. H. Hyers, *On the stability of the linear functional equation,* Proc. Nat. Acad. Sci. USA., **27**(1941), 222-224.

[17] M. Kuczma,*An introduction to the theory of functional equations and inequalities: Cauchy's equation and Jensen's inequality*, Birkhauser Verlag, Basel, 2009.

[18] S. Lee, S. Im and I. Hwang, *Quartic functional equations*, J. Math. Anal. Appl., **307**(2005), 387–394.

[19] C. Park, A. Bodaghi and T.-Z. Xu, *On an equation characterizing multi-Jensen-quartic mappings and its stability*, J. Math. Inequal., **15**(1)(2021), 333-347.

[20] C.-G. Park, *Multi-quadratic mappings in Banach spaces*, Proc. Amer. Math. Soc., **131**(2002), 2501-2504.

[21] J. M. Rassias, *Solution of the Ulam stability problem for quartic mappings*, Glasnik Mat. Ser. III, **34**(2)(1999), 243–252.

[22] J. M. Rassias, M. Arunkumar and E.Sathya, *Non-stabilities of mixed type Euler-Lagrange k-cubic-quartic functional equation in various normed spaces*, Math. Anal. Cont. Appl., **1**(1)(2019), 1-43.

[23] J. M. Rassias, E. Sathya and M. Arunkumar, *Generalized Ulam-Hyers stability of an alternate additive-quadratic-quartic functional equation in fuzzy Banach spaces*, Math. Anal. Cont. Appl., **3**(1)(2021), 13-31.

[24] Th. M. Rassias, *On the stability of the linear mapping in Banach spaces*, Proc. Amer. Math. Soc., **72**(2)(1978), 297-300.

[25] S. M. Ulam, *Problems in modern mathematics,* Chapter VI, Science Ed., Wiley, New York, 1940.

[26] X. Zhao, X. Yang and C.-T. Pang, *Solution and stability of the multiquadratic functional equation*, Abstr. Appl. Anal., **2013**(2013), Art. ID 415053.

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